

A Constrained Iterative LQR Solver for the Trajectory Optimization Framework Horizon

Arturo Laurenzi, Francesco Ruscelli, and Nikos G. Tsagarakis



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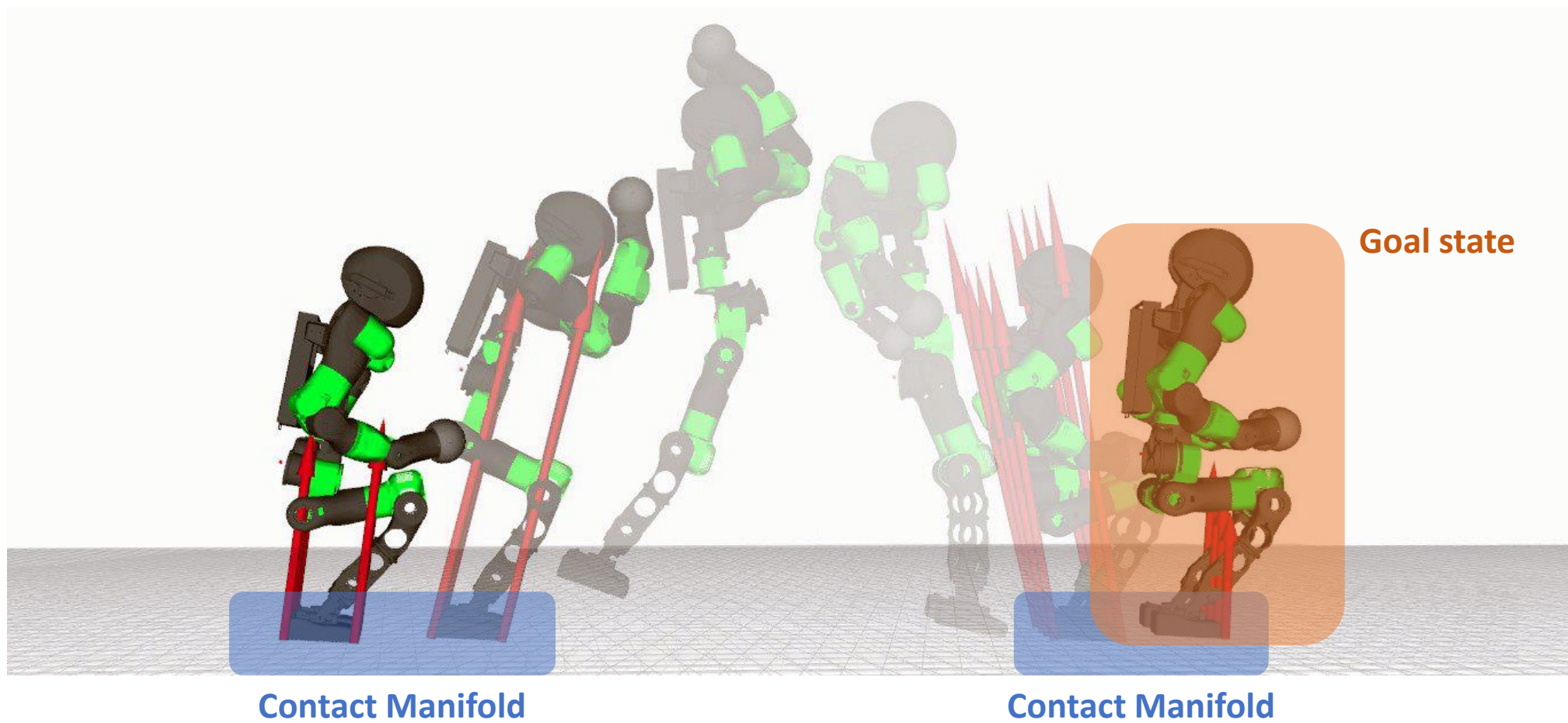
The eLQR problem

$$\begin{aligned} \min_{x_{0:N}, u_{0:N-1}} \quad & \sum_{k=0}^{N-1} \ell_k(x_k, u_k) + \ell_N(x_N) \\ \text{s.t.} \quad & x_{k+1} = F(x_k, u_k) \\ & h_k(x_k, u_k) = 0, \quad h_N(x_N) = 0, \end{aligned}$$

Path vs waypoint constraints

- **Path constraints** keep the system state-input on a manifold
 - Constraint dimension smaller than input dimension
 - Easily dealt with via **projection** (null-space) methods
 - Example: the contact manifold
- **Waypoint constraints** (e.g. final constraints) are useful to specify a goal
 - Constraint dimension up to the state dimension
 - **Cannot be fulfilled with the choice of a single control input !**
 - Example: posture at the end of a jump motion
 - Example: space travelled after taking N steps

An example



Our contribution

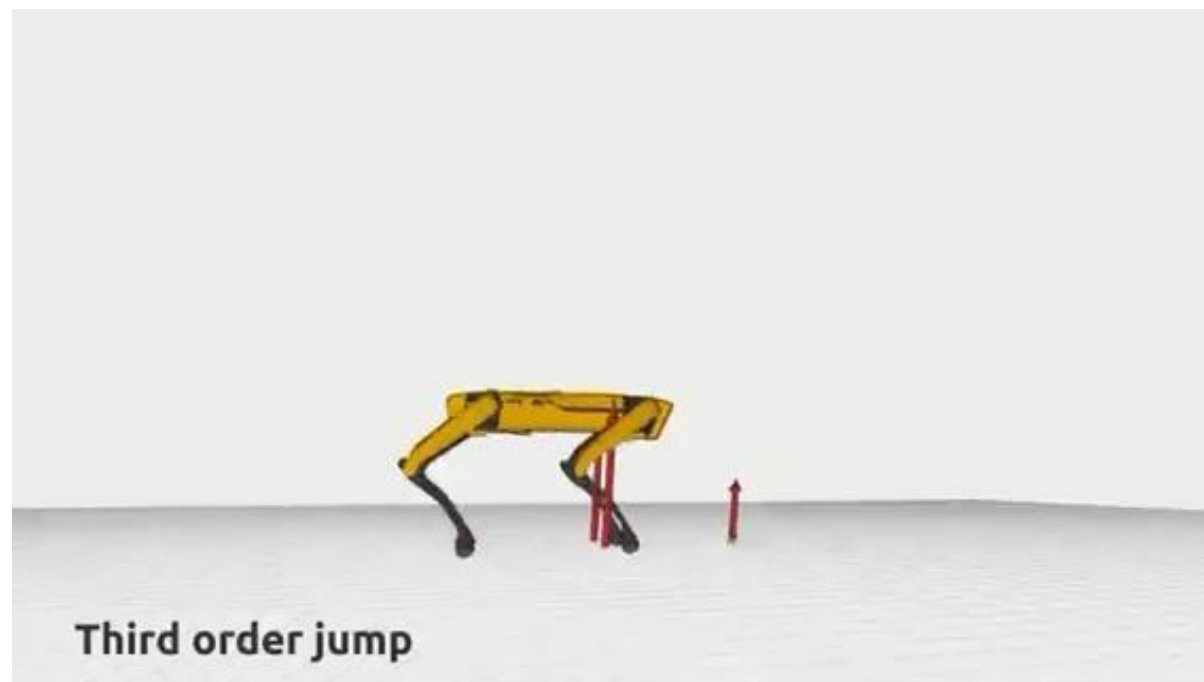
- A **Riccati**-like recursion to compute
 - The eLLQR **optimal policy**
 - **Lagrange multiplier** estimates
- Lagrange multipliers are useful to **autotune** an L1 merit function

$$m(X) = L(X) + \gamma \|H(X)\|_1$$

- An open-source implementation
- Extensive validation campaign



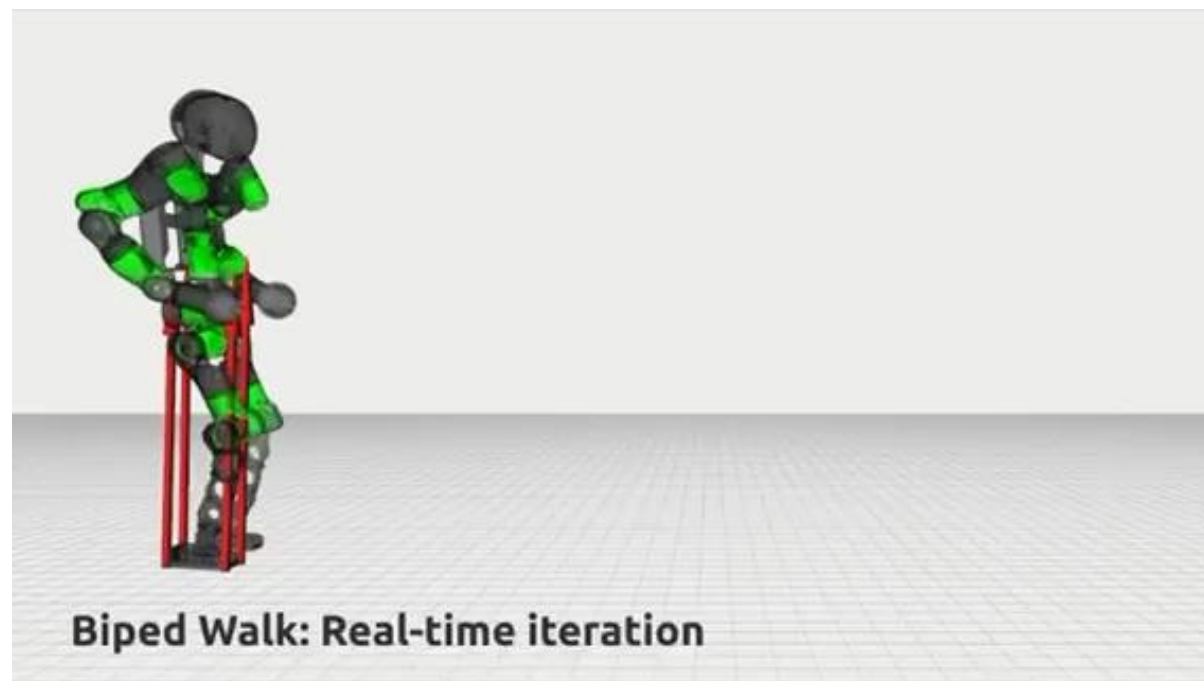
Centauro Walk: Climb Off



Third order jump



Centauro Walk: Real-time Iteration



Biped Walk: Real-time iteration

Meet me at
the poster
session !

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1. Contribution at a glance

- An ILQR algorithm for general **equality-constrained** problems
- **O(N)** complexity w.r.t. horizon length
- Computes a **linear policy** for both the control input δu and Lagrange multipliers $\delta \mu$, $\delta \lambda$
- Exploit Lagrange multipliers estimate to implement an **exact L1 line search** strategy
- Extensive **validation** on complex robotic examples

2. Problem definition

A **discrete-time** Trajectory Optimization (TO) problem, with **equality constraints**

$$\begin{aligned} \min_{x_{0:N}, u_{0:N-1}} \quad & \sum_{k=0}^{N-1} \ell_k(x_k, u_k) + \ell_N(x_N) \\ \text{s.t.} \quad & x_{k+1} = F(x_k, u_k) \\ & h_k(x_k, u_k) = 0, \quad h_N(x_N) = 0 \end{aligned}$$

Note the final constraint cannot be dealt with via projection/nullspace approaches!

3. Approach outline

Our strategy

- Apply **Newton's method** to the KKT conditions for the TO problem
- Solve the resulting linear system with **Riccati-like recursions** (backward pass + forward pass)

1) Hypothesize the following relation hold at node k

$$\begin{aligned} S_{k+1} \delta x_{k+1} + V_{k+1}^T \delta u_{k+1} - \delta \lambda_k &= -s_{k+1} \\ V_{k+1} \delta x_{k+1} &= -v_{k+1} \end{aligned}$$

show that it holds at $k-1$, too.

2) Back-propagate constraint via the dynamics

$$C_k \delta x_k + D_k \delta u_k = c_k$$

3) Handle rank-deficiency of D_k . A generic state-only constraint cannot be solved by a single control input!

- Separate feasible-infeasible components at time k
- Do it also for Lagrangian multipliers

4. Globalization strategy

Promote convergence to a **local minimum** by enforcing the decrease of a **merit function**

$$m(X) = L(X) + \gamma \|H(X)\|_1,$$

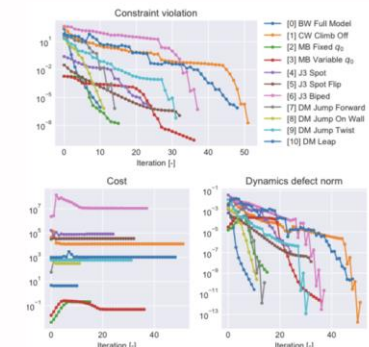
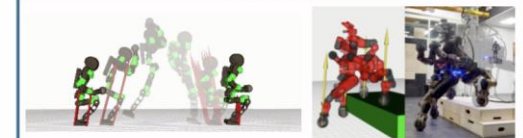
The merit function $m(X)$ is **exact**^a if

$$\gamma > \max\{\|\lambda_{0:N-1}^*\|_\infty, \|\mu_{0:N}^*\|_\infty\}$$

We can exploit the computed Lagrangian multiplier estimates to **tune γ automatically**

^aA merit function is said to be **exact** if its local minima are also local minima for the original constrained problem.

5. Validation



- Behaviors entirely obtained via **constraints**
- Contact model, centroidal dynamics enforced via **constraints**



Acknowledgements



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