Optimal Scheduling of Models and Horizons for Model Hierarchy Predictive Control

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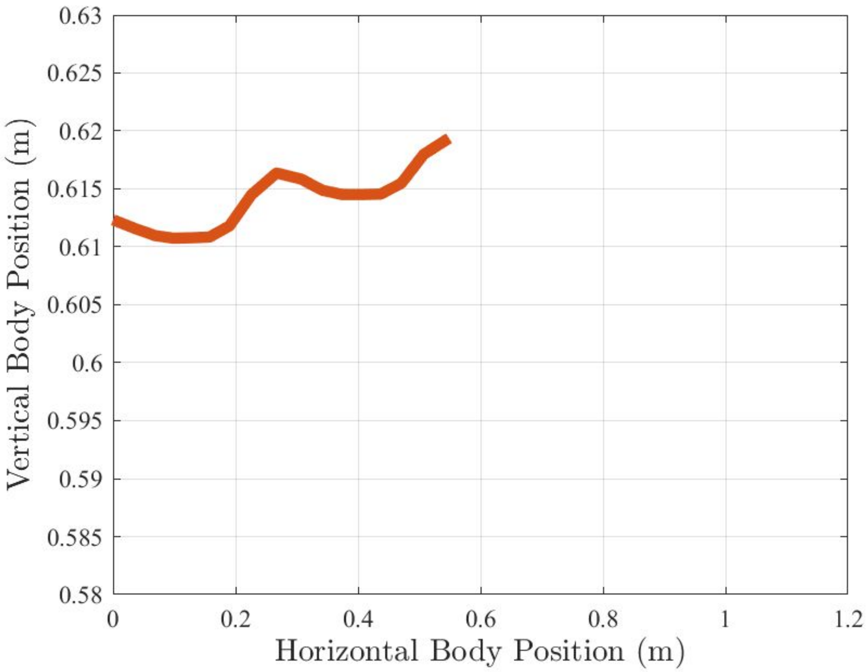
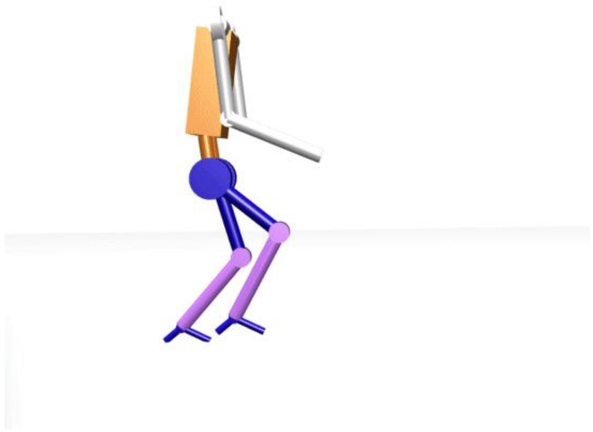
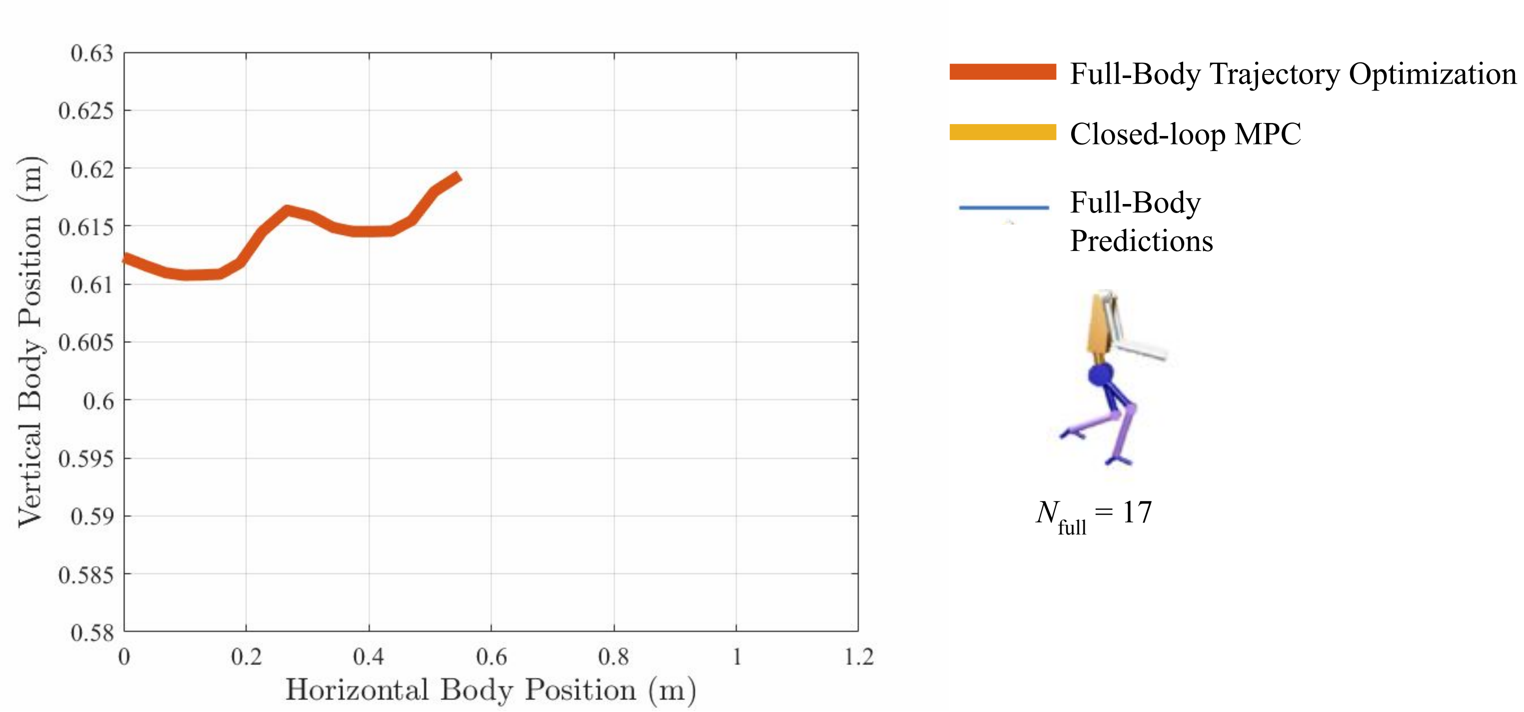
Trajectory Optimization



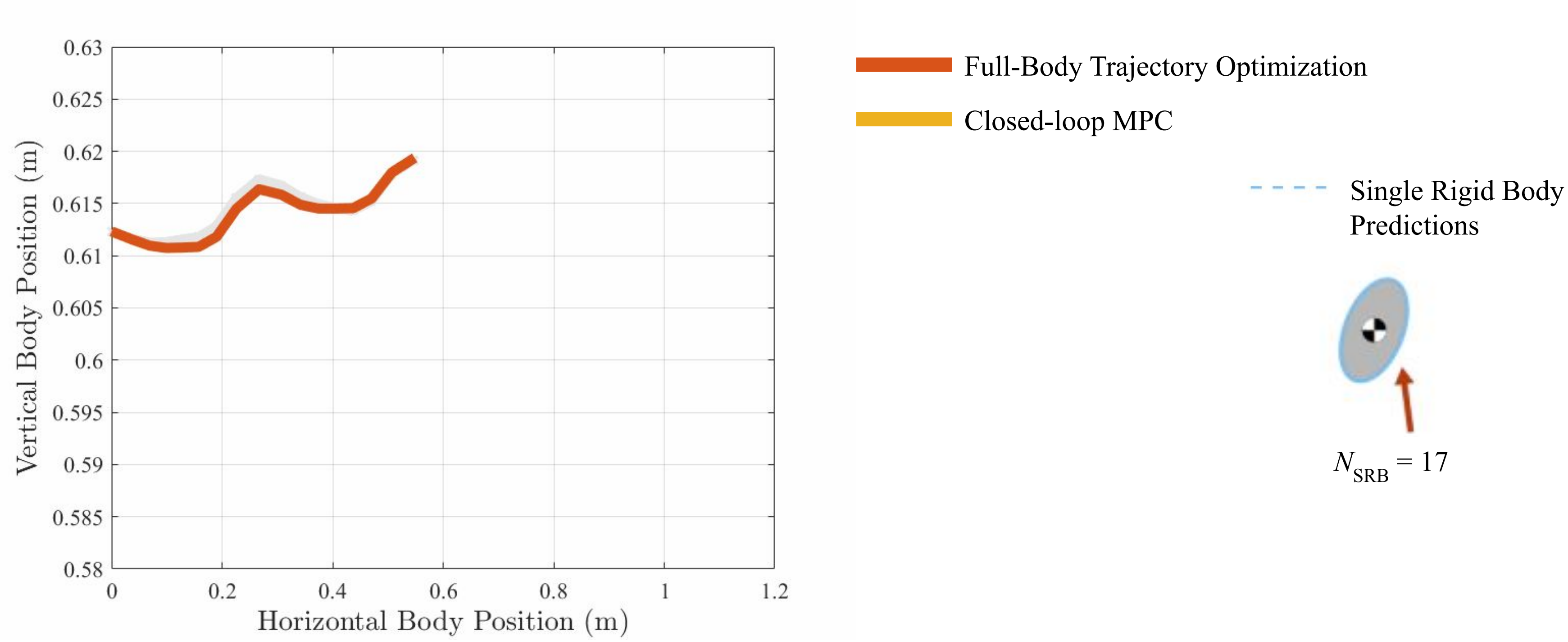
Full-Body Trajectory Optimization



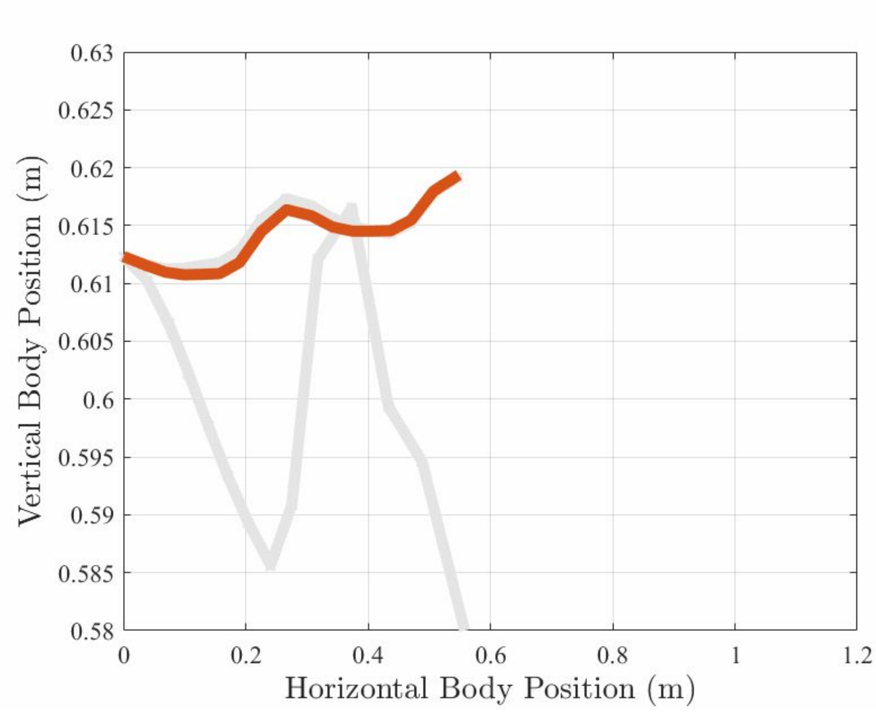
Model Predictive Control

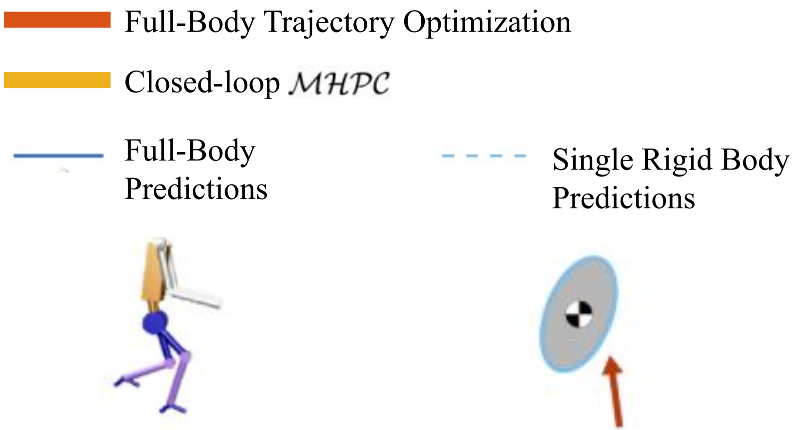


Model Predictive Control



Model Hierarchy Predictive Control





$N_{\mathrm{full}} = 9$ $N_{\mathrm{SRB}} = 8$

We present systematic approach to optimize the model schedule

H. Li, R. J. Frei and P. M. Wensing, "Model Hierarchy Predictive Control of Robotic Systems", 2021

Model Schedule Optimization

$\boldsymbol{\mathcal{S}}$: model schedule

$\hat{\boldsymbol{\mathcal{X}}}$: Set of decision variables

$V^{\boldsymbol{\mathcal{S}}}$: Closed-loop Cost of $\mathcal{MHPC}$

V^*: Optimal Cost

$$\boldsymbol{\mathcal{S}} = \boldsymbol{\mathcal{S}}^* = \arg\min_{\boldsymbol{\mathcal{S}}} |\hat{\boldsymbol{\mathcal{X}}}|$$

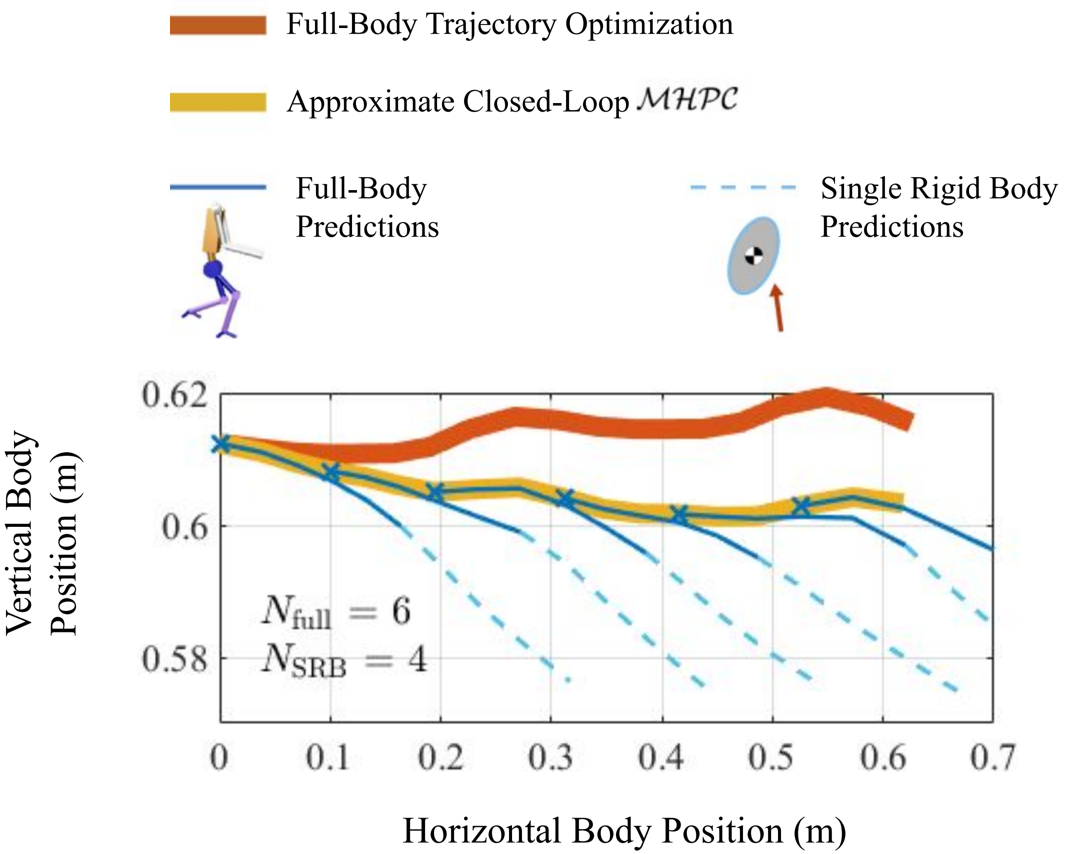
Minimize the number of decision variables

subject to

$$\frac{V^{\boldsymbol{\mathcal{S}}}(\boldsymbol{x}_1) - V^*(\boldsymbol{x}_1)}{V^*(\boldsymbol{x}_1)} \leq \epsilon$$

Closed-loop cost near optimal

Model Schedule Optimization



$$V^*(\boldsymbol{x}_1) \leq \epsilon$$

From Full-Body to Reduced Order Dynamics

$$\boldsymbol{H}(\boldsymbol{q}_k)(\dot{\boldsymbol{q}}_{k+1} - \dot{\boldsymbol{q}}_k) + \boldsymbol{C}(\boldsymbol{q}_k, \dot{\boldsymbol{q}}_k)dt_k = \boldsymbol{B}\boldsymbol{u}_k dt_k$$

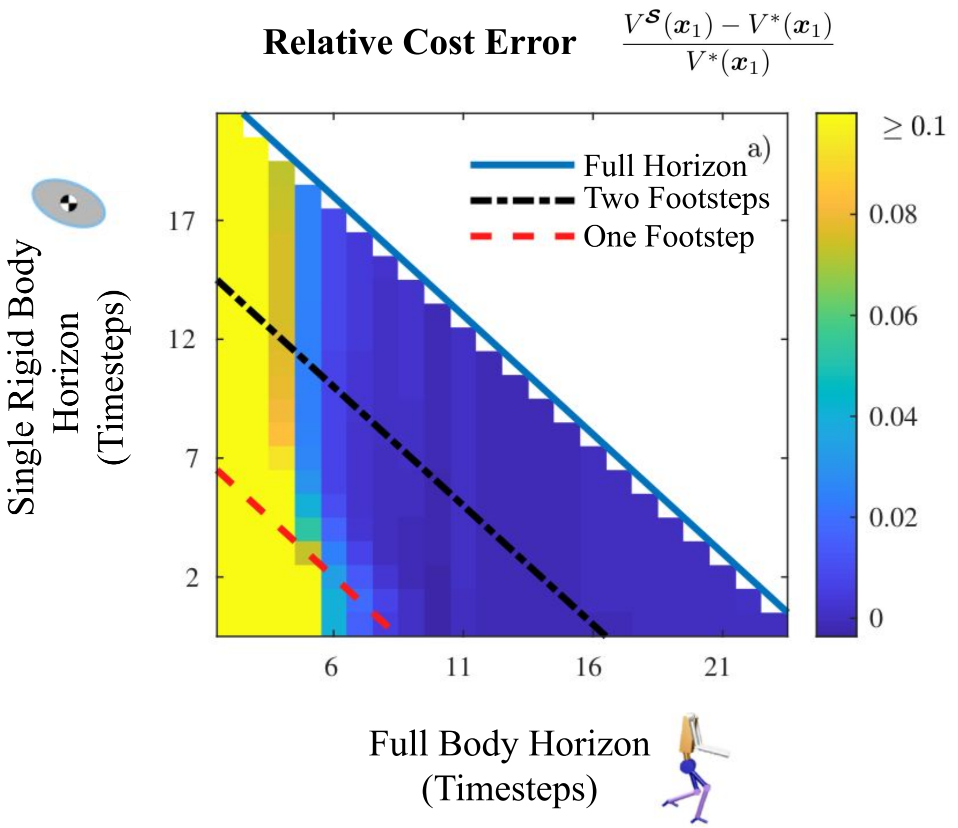
$$+ s_{\mathrm{full},k}\boldsymbol{J}_c^\top(\boldsymbol{q}_k)\boldsymbol{F}_{c,k}dt_k$$

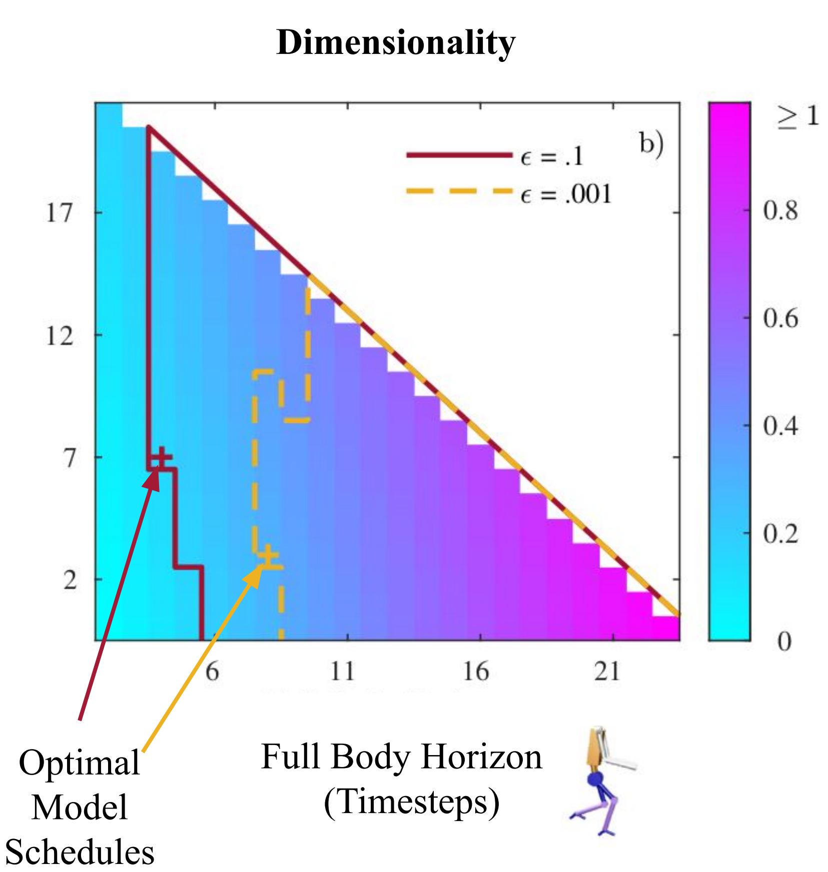
$$+ s_{\mathrm{SRB},k}\boldsymbol{J}_{\mathrm{SRB}}^\top(\boldsymbol{q}_k)\boldsymbol{F}_{\mathrm{SRB},k}dt_k$$

$$+ s_{\mathrm{SRB},k}\boldsymbol{J}_{\mathrm{fb}}^\top\boldsymbol{F}_c dt_k,$$

$$+ s_{\mathrm{void},k}\boldsymbol{J}_{\mathrm{void}}^\top(\boldsymbol{q}_k)\boldsymbol{F}_{\mathrm{void},k}dt_k$$

Cost vs Dimensionality





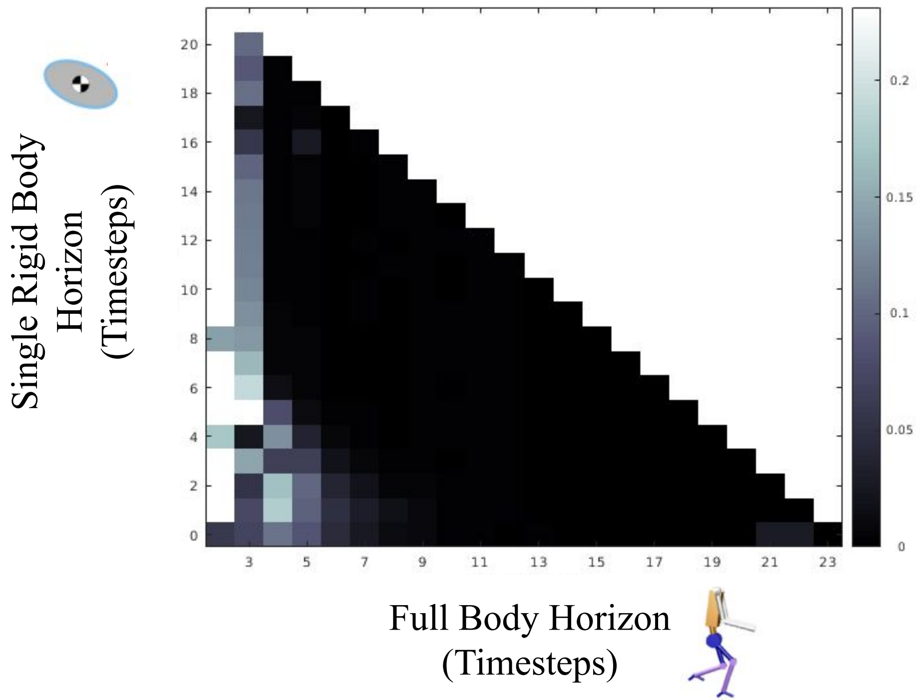
Trajectory Optimization Approximates Closed-Loop Costs

$$\left.\frac{V^{\mathcal{S}}(\boldsymbol{x}_1) - V^*(\boldsymbol{x}_1)}{V^*(\boldsymbol{x}_1)}\right|_{\text{TO}}$$

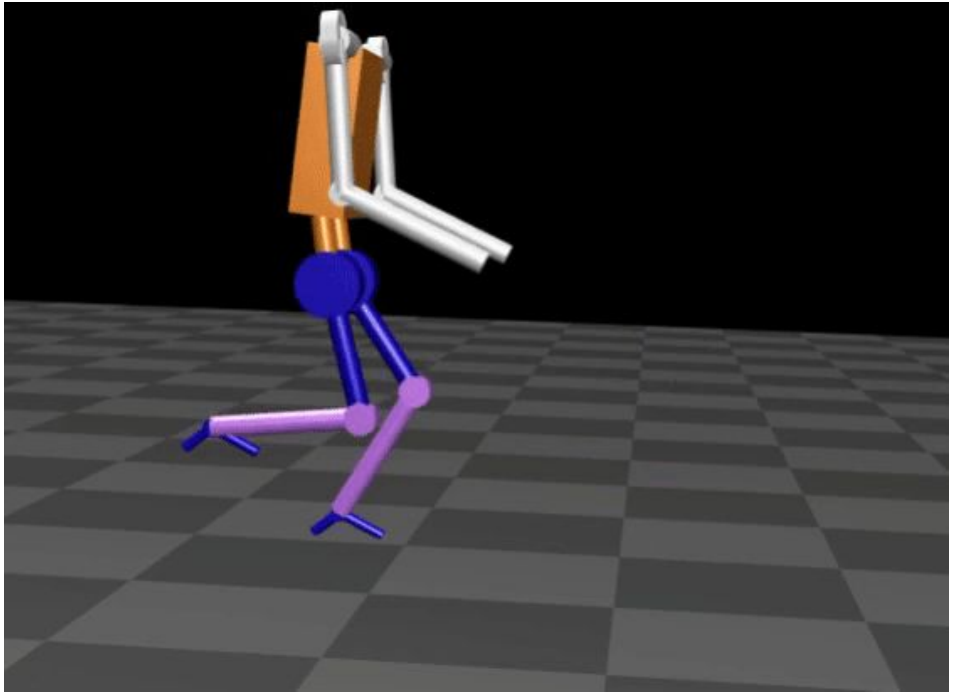
VS

$$\left.\frac{V^{\mathcal{S}}(\boldsymbol{x}_1) - V^*(\boldsymbol{x}_1)}{V^*(\boldsymbol{x}_1)}\right|_{\text{Sim}}$$

Normalized Cost Error Difference (Our approach - Simulation)



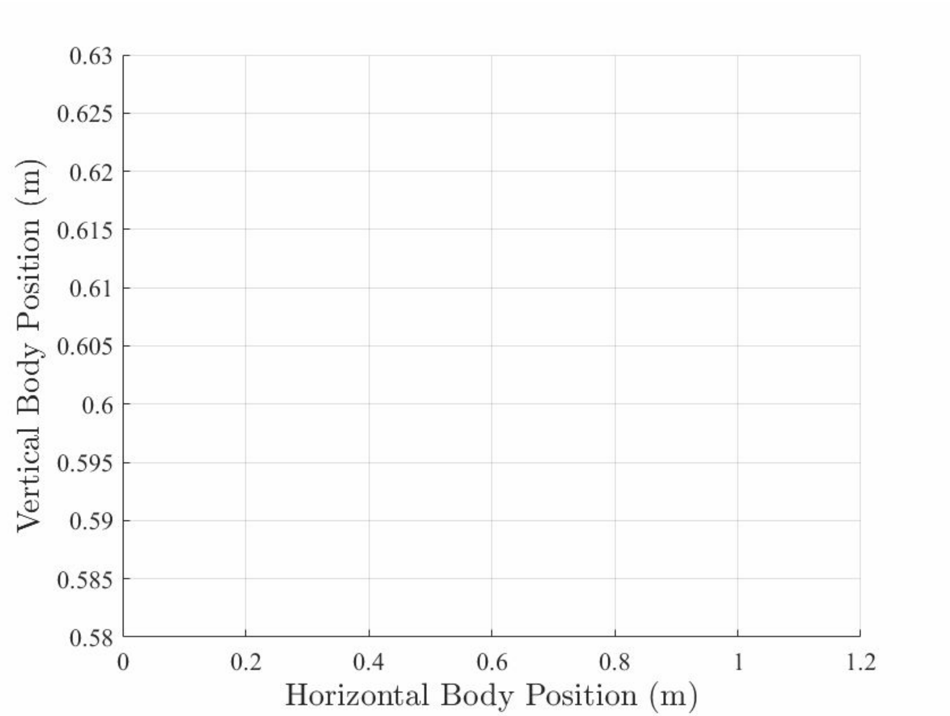
From Full-Body to Reduced Order Dynamics

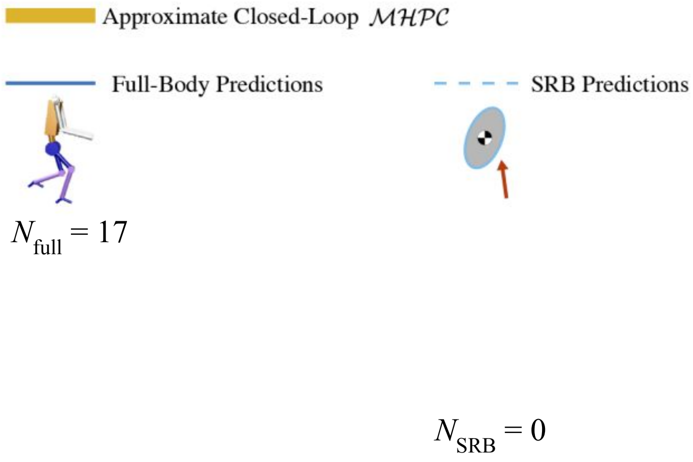


Future Work

- In this work, proof-of-concept with a small search space
- Use of more efficient algorithms
    - Relax the integer variable + rounding (only if tight)
    - Mixed-integer non-linear programming leveraging branch and bound algorithms
    - Gradient-free algorithms like particle swarm optimization

Model Predictive Control





Model Predictive Control

$$V^*(\boldsymbol{x}_1) = \min_{\boldsymbol{\mathcal{X}}} \sum_{k=1}^{N-1} c(\boldsymbol{x}_k, \boldsymbol{u}_k) + c_N(\boldsymbol{x}_N) \qquad (\mathcal{TO})$$

subject to

$$\boldsymbol{x}_{k+1} = \boldsymbol{f}(\boldsymbol{x}_k, \boldsymbol{u}_k) \; \forall \; k \in \{1, ..., N-1\}$$
$$\boldsymbol{g}(\boldsymbol{x}_k, \boldsymbol{u}_k) \leq \boldsymbol{0} \; \forall \; k \in \{1, ..., N-1\}$$
$$\boldsymbol{g}_N(\boldsymbol{x}_N) \leq \boldsymbol{0},$$

$$\hat{V}^{\boldsymbol{\mathcal{S}}}(\hat{\boldsymbol{x}}_1) = \min_{\hat{\boldsymbol{\mathcal{X}}}} \sum_{k=1}^{\hat{N}-1} \hat{c}_k(\hat{\boldsymbol{x}}_k, \hat{\boldsymbol{u}}_k) + \hat{c}_{\hat{N}}(\hat{\boldsymbol{x}}_{\hat{N}}) \qquad (\mathcal{MHPC})$$

subject to

$$\hat{\boldsymbol{x}}_{k+1} = \hat{\boldsymbol{f}}_k(\hat{\boldsymbol{x}}_k, \hat{\boldsymbol{u}}_k) \; \forall \; k \in \{1, ..., \hat{N}-1\}$$
$$\hat{\boldsymbol{g}}_k(\hat{\boldsymbol{x}}_k, \hat{\boldsymbol{u}}_k) \leq \boldsymbol{0} \; \forall \; k \in \{1, ..., \hat{N}-1\}$$
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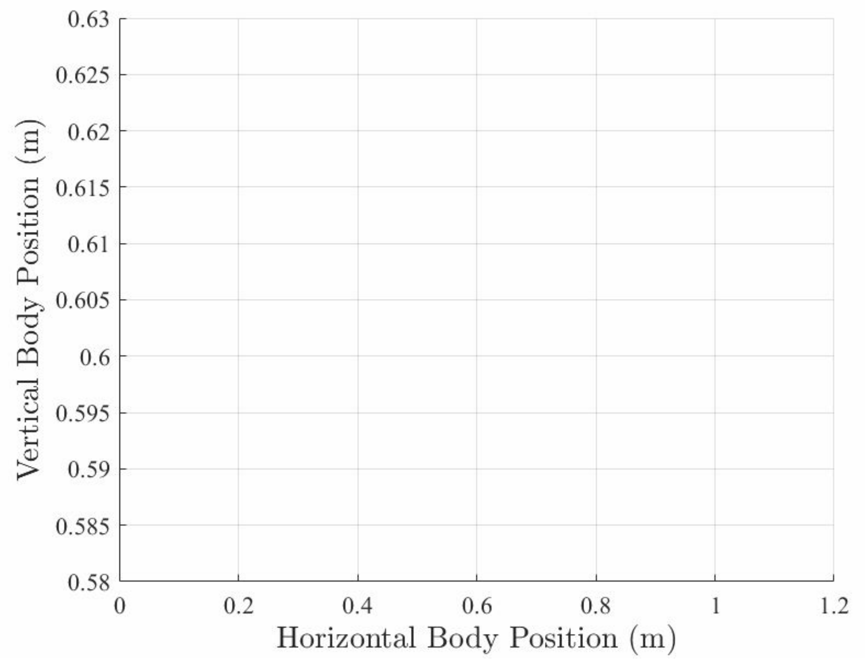
Model Predictive Control

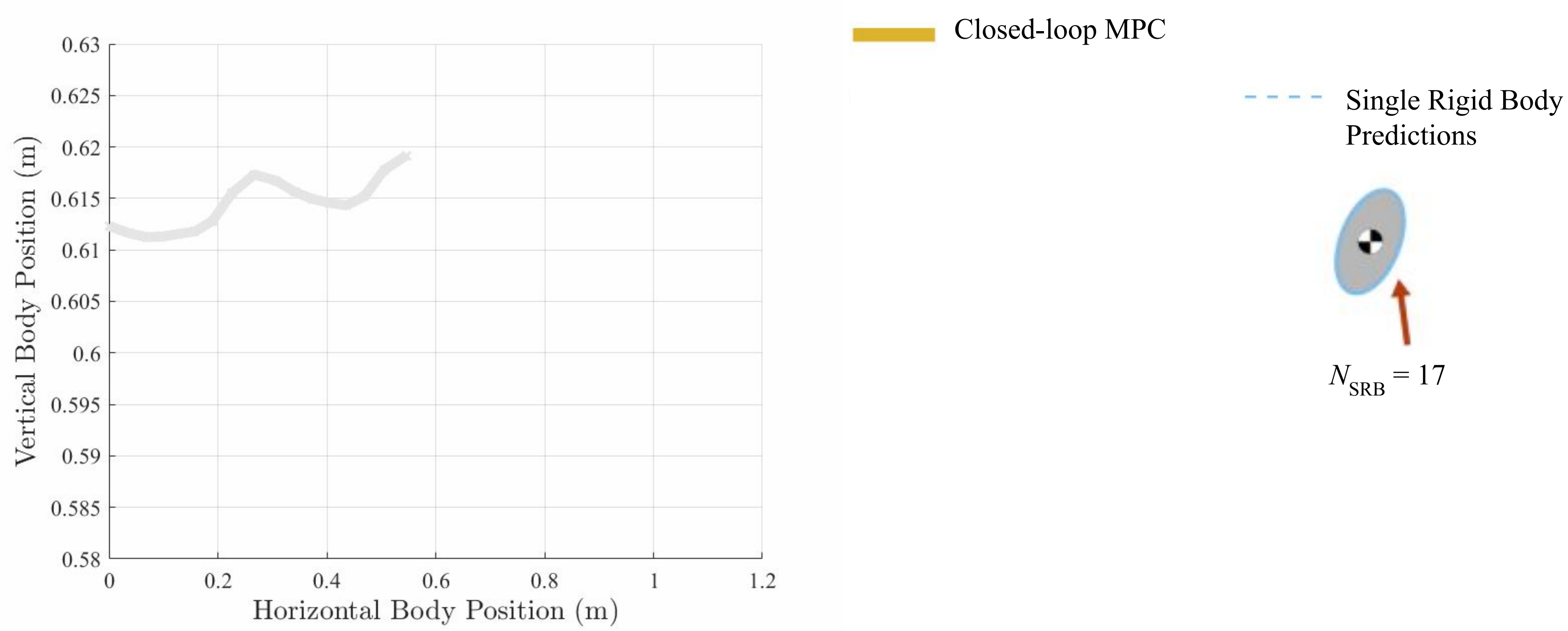


Closed-loop MPC

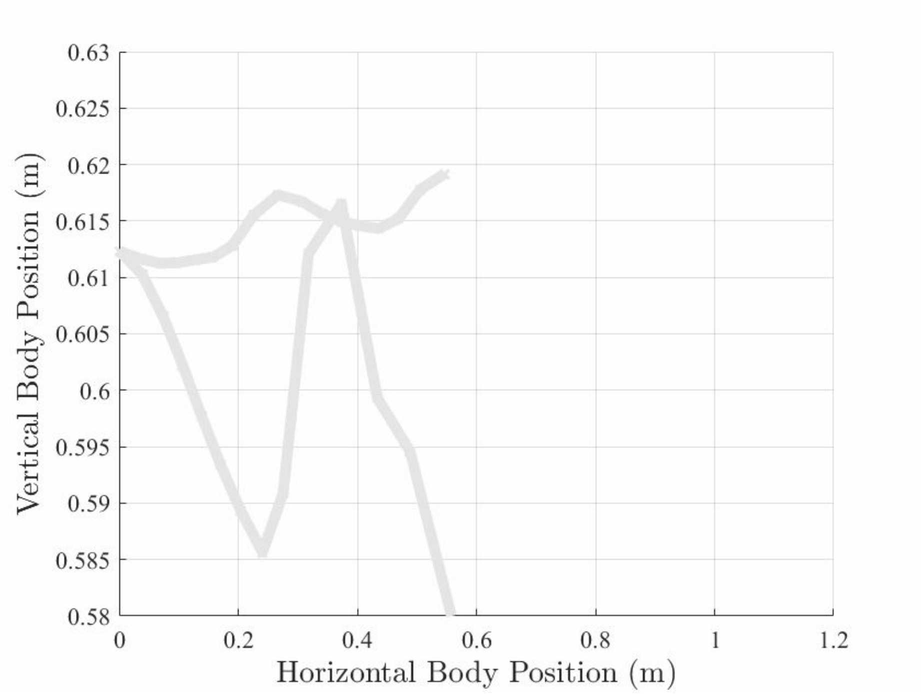
Full-Body Predictions

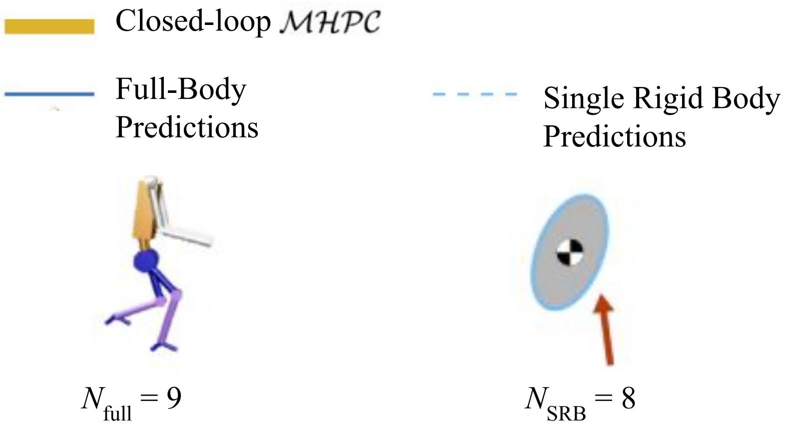
$N_{\text{full}} = 17$

Model Predictive Control



Model Hierarchy Predictive Control





How to optimize the model schedule ?

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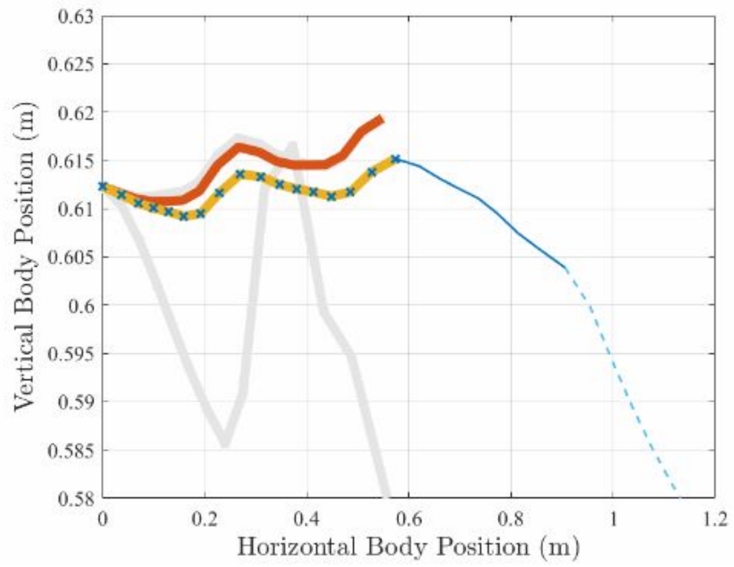

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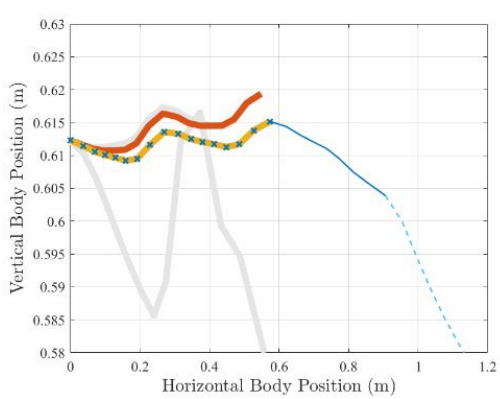
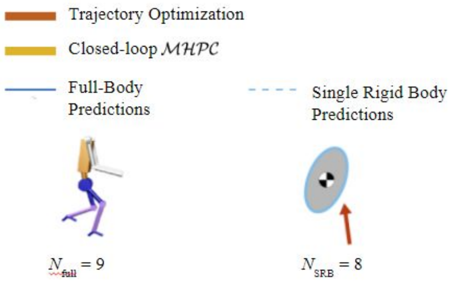
$V^{\mathcal{S}}$: Closed-loop Cost of $\mathcal{MHPC}$





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Minimize the number of decision variables

Closed-loop cost near optimal