

[Humanoids 2022 Workshop] - Advancements in Trajectory Optimization and Model Predictive Control for Legged Systems

**frontiers**Frontiers in **Robotics and AI**

# Adaptive robot climbing with magnetic feet in unknown slippery structures

Jee-eun Lee <sup>1</sup>, Tirthankar Bandyopadhyay <sup>2</sup>, and Luis Sentis <sup>1</sup>

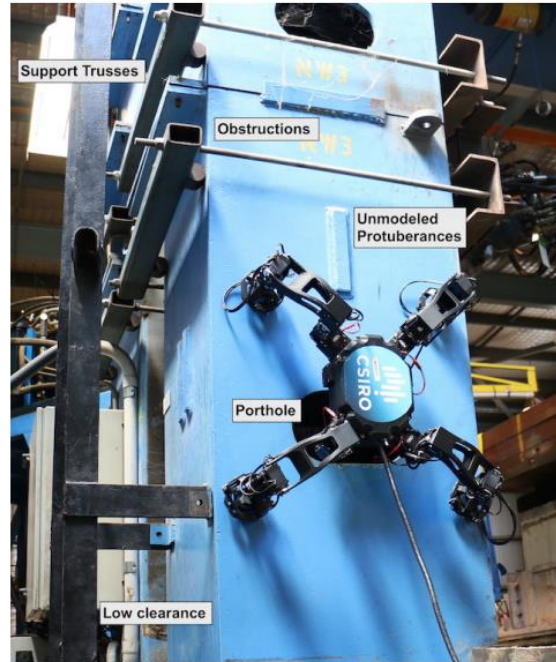
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**TEXAS**  
The University of Texas at Austin

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Systems Research Group, CSIRO

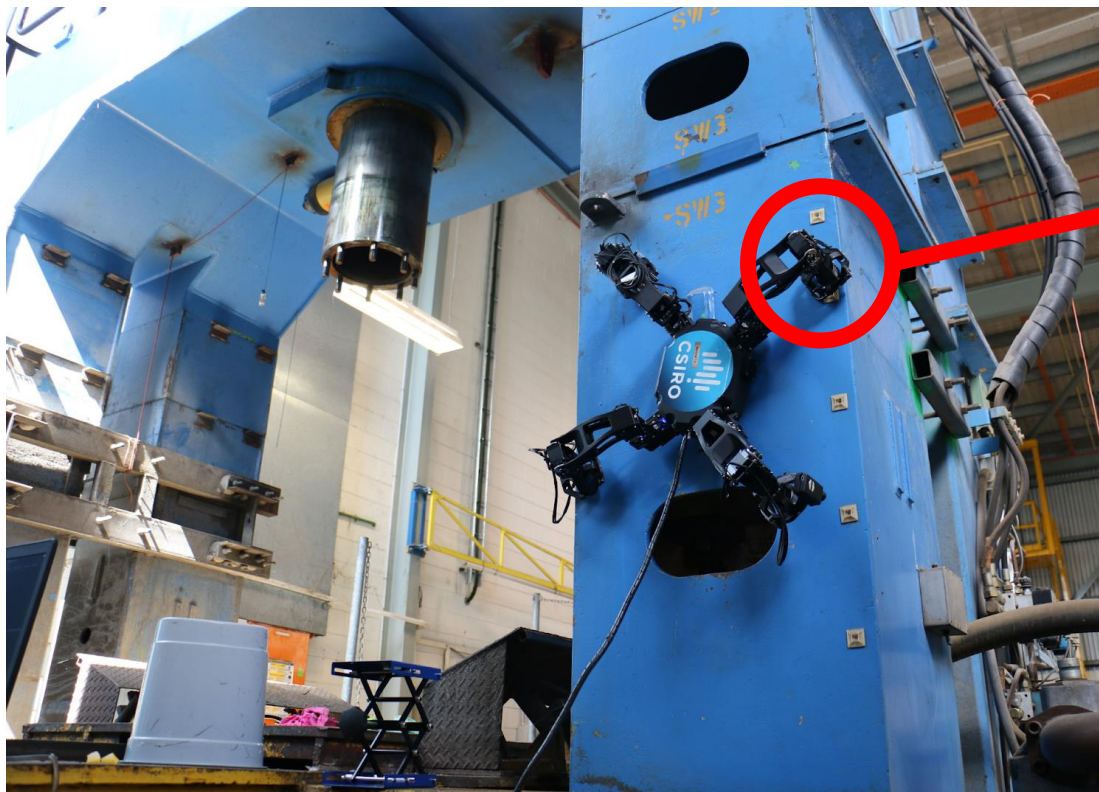


# Climbing robots to reduce human risks at height\*



\* Bandyopadhyay, Tirthankar, et al. "Magneto: A versatile multi-limbed inspection robot." *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2018.

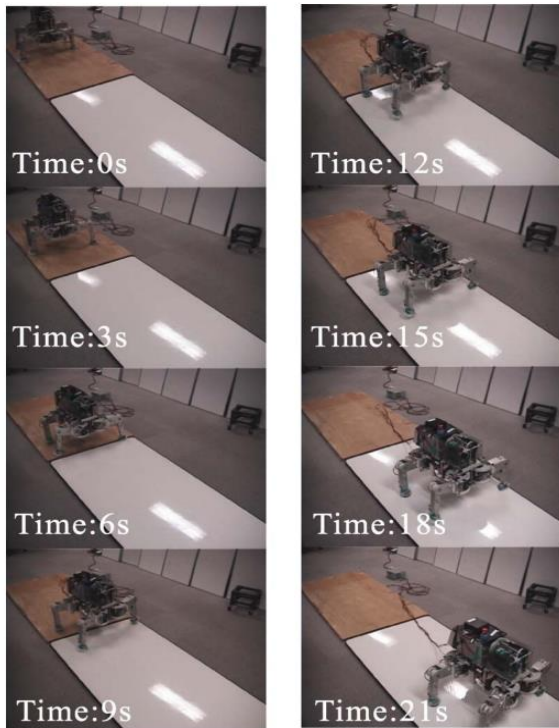
However, unknown slippery conditions can cause problems



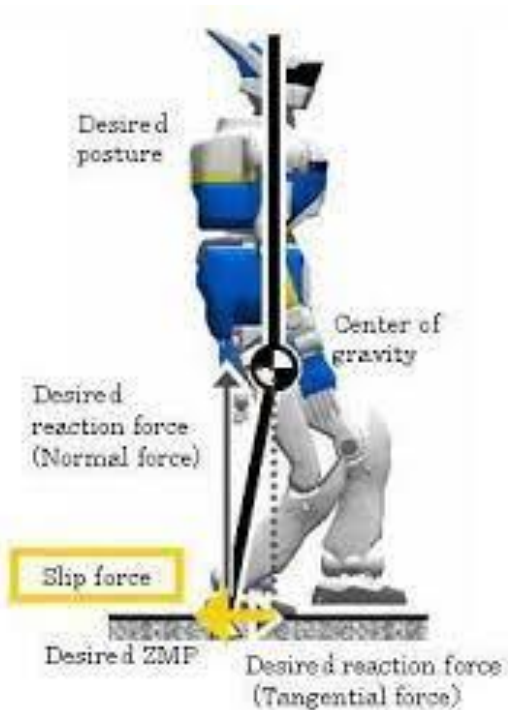
friction coefficient  $\mu$   
(magnetic) adhesive force  $F_m$

...

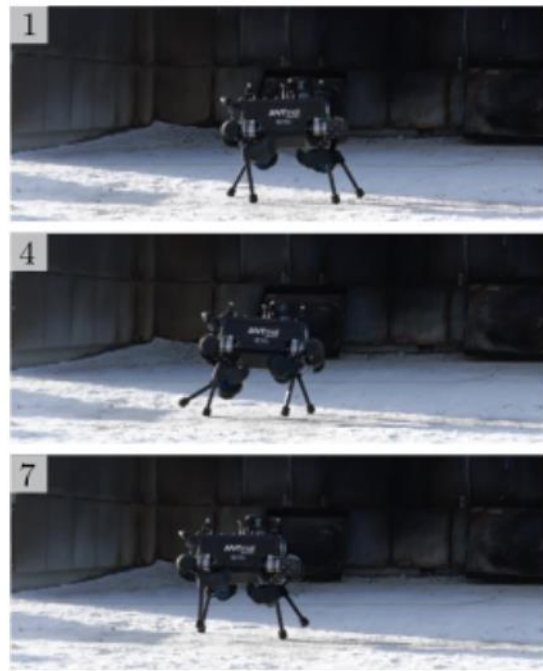
# Related Works / Limitations



(Takemura et al. (2005))



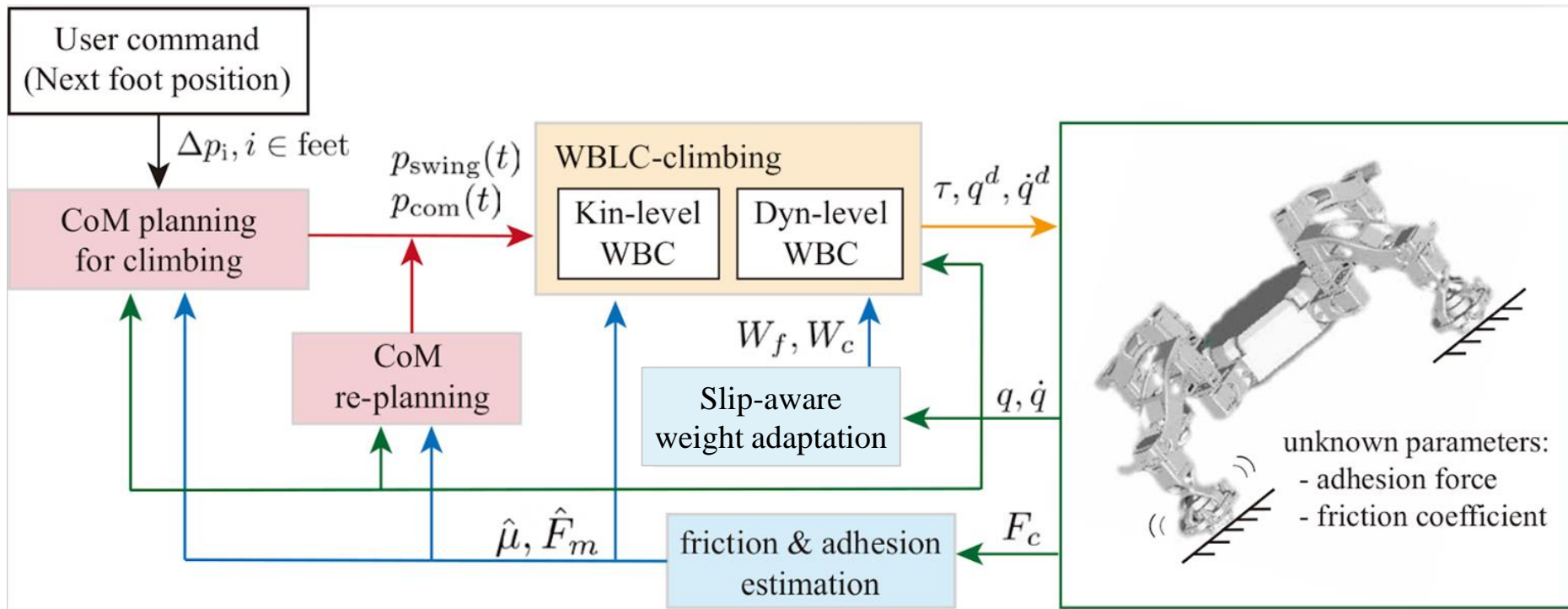
(Kaneko et al. (2005))



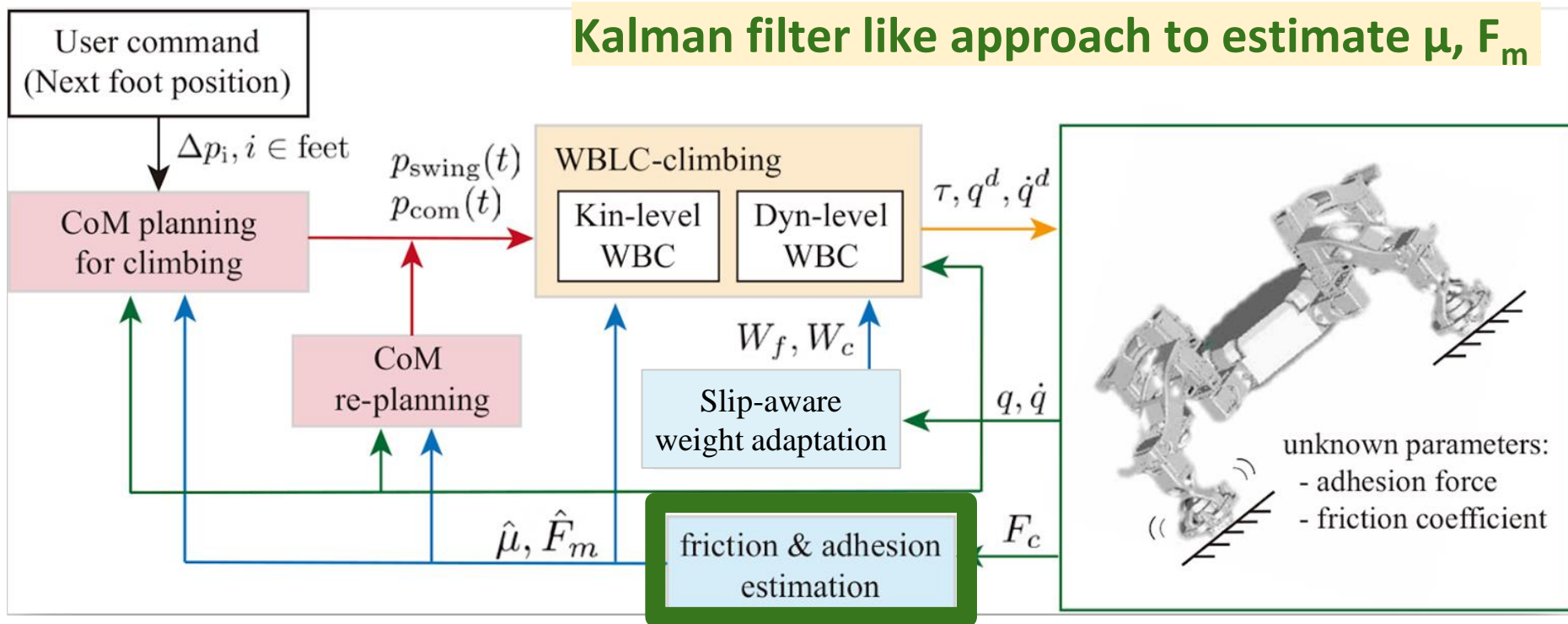
(Jenelten et al. (2019))



# The Proposed Control Architecture

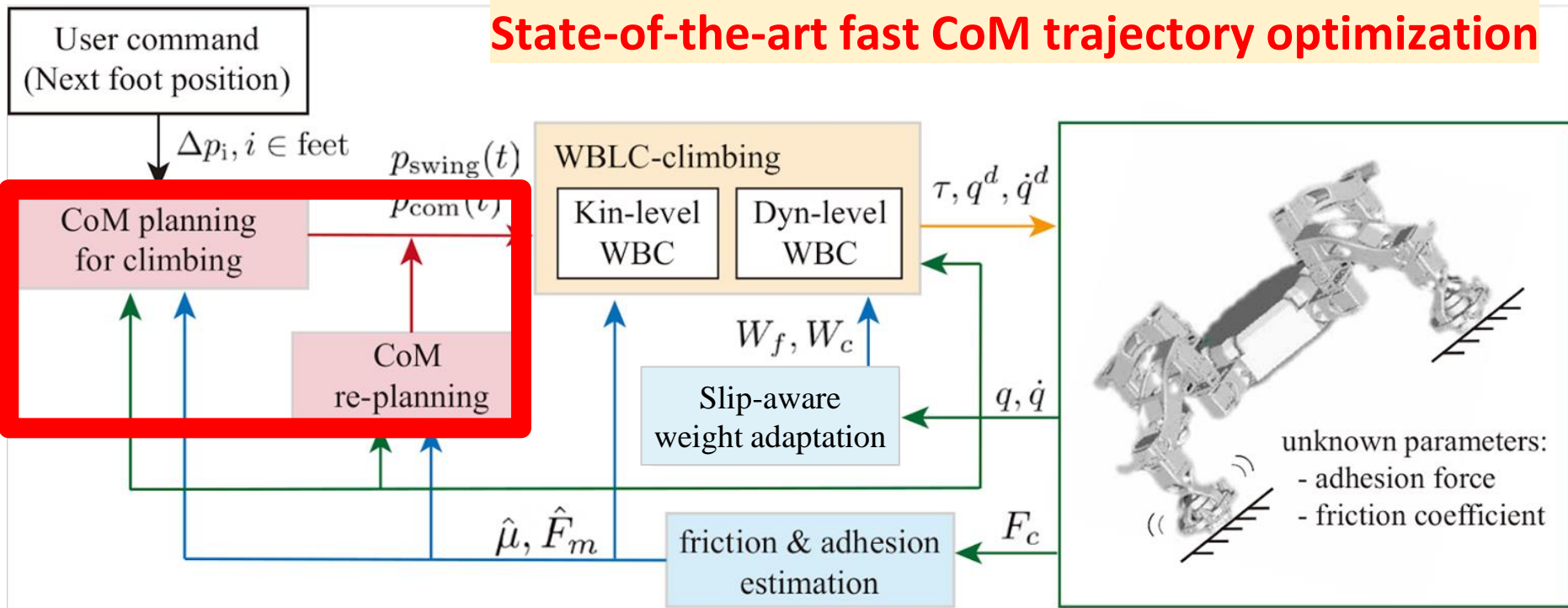


# The Proposed Control Architecture

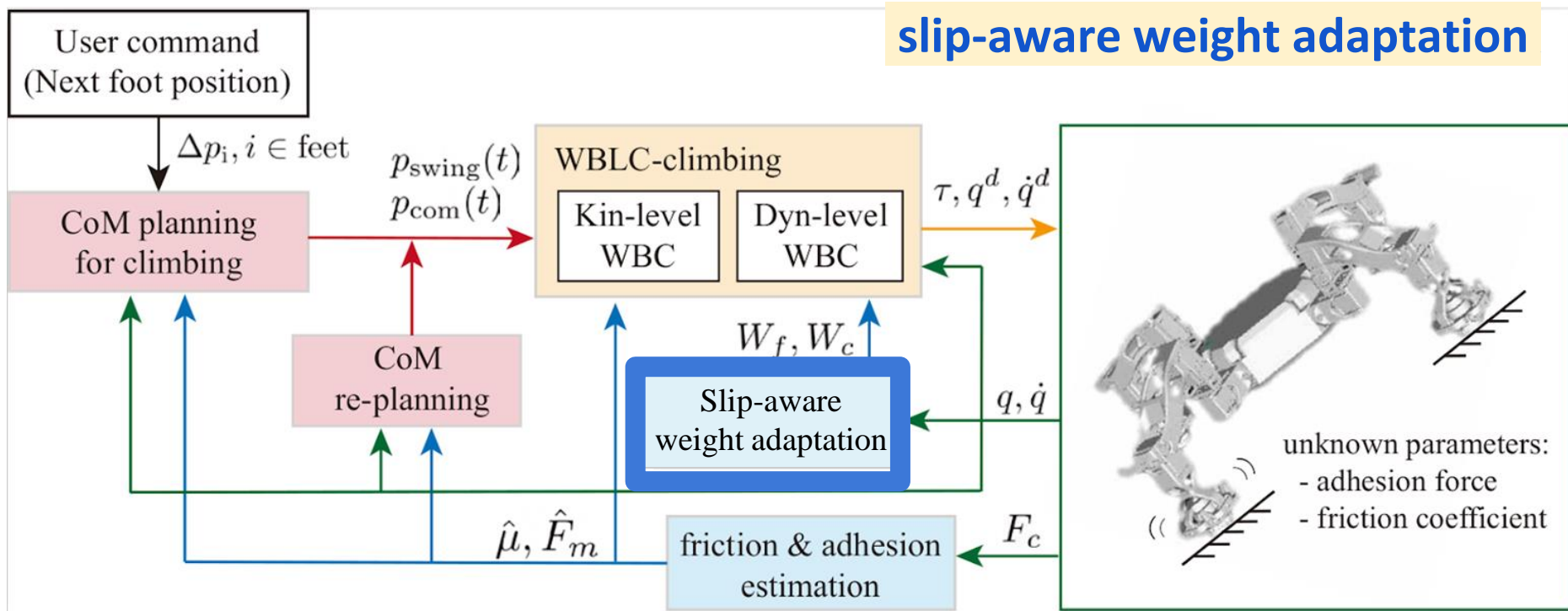


# The Proposed Control Architecture

## State-of-the-art fast CoM trajectory optimization

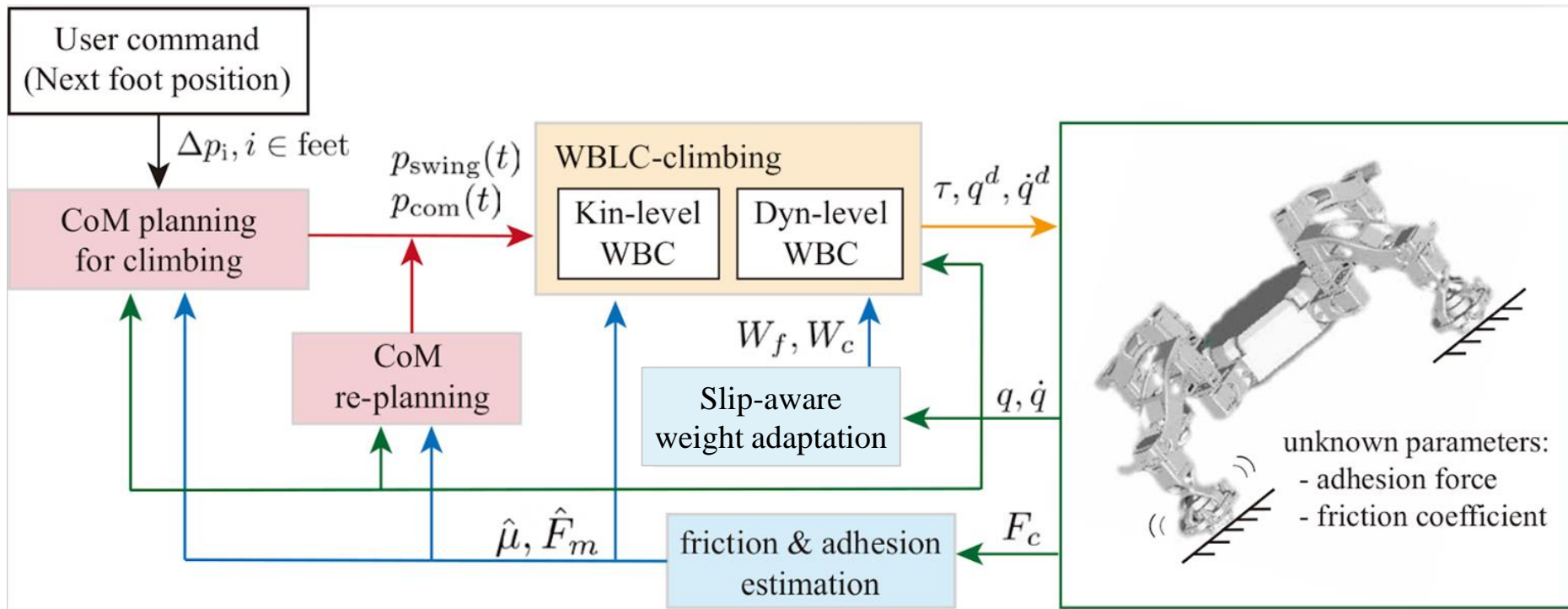


# The Proposed Control Architecture





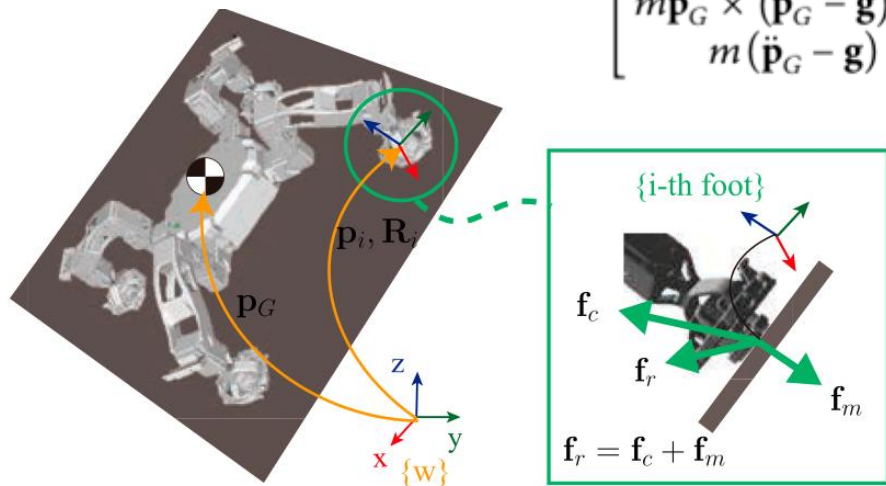
# The Proposed Control Architecture



# **(i) Multi-contact CoM trajectory generation for climbing**

# Multi-contact CoM trajectory generation for climbing

- Centroidal dynamics with magnetic force



$$\begin{aligned}
 \begin{bmatrix} m\mathbf{p}_G \times (\ddot{\mathbf{p}}_G - \mathbf{g}) + \dot{\mathbf{L}} \\ m(\ddot{\mathbf{p}}_G - \mathbf{g}) \end{bmatrix} &= \sum_{i \in \text{Contact}} \begin{bmatrix} \mathbf{p}_i \times \mathbf{R}_i \mathbf{f}_{c,i} \\ \mathbf{R}_i \mathbf{f}_{c,i} \end{bmatrix} + \sum_{i \in \text{Magnet}} \begin{bmatrix} \mathbf{p}_i \times \mathbf{R}_i \mathbf{f}_{m,i} \\ \mathbf{R}_i \mathbf{f}_{m,i} \end{bmatrix}, \\
 &= \underbrace{\begin{bmatrix} \mathbf{p}_{c1} \times \mathbf{R}_{c1} \cdots \\ \mathbf{R}_{c1} \cdots \end{bmatrix}}_{\mathcal{P}_c} \underbrace{\begin{bmatrix} \mathbf{f}_{c,c1} \\ \vdots \end{bmatrix}}_{\mathbf{f}_c} + \underbrace{\begin{bmatrix} \mathbf{p}_{m1} \times \mathbf{R}_{m1} \cdots \\ \mathbf{R}_{m1} \cdots \end{bmatrix}}_{\mathcal{P}_m} \underbrace{\begin{bmatrix} \mathbf{f}_{m,m1} \\ \vdots \end{bmatrix}}_{\mathbf{f}_m} \\
 &= \begin{bmatrix} \mathcal{P}_c \\ \mathcal{R}_c \end{bmatrix} \underbrace{\mathbf{f}_c}_{\text{blue circle}} + \begin{bmatrix} \mathcal{P}_m \\ \mathcal{R}_m \end{bmatrix} \underbrace{\mathbf{f}_m}_{\text{blue circle}} \quad (5)
 \end{aligned}$$

- Friction cone constraints  $\mathbf{D}\mathbf{f}_c \geq 0,$

# Multi-contact CoM trajectory generation for climbing

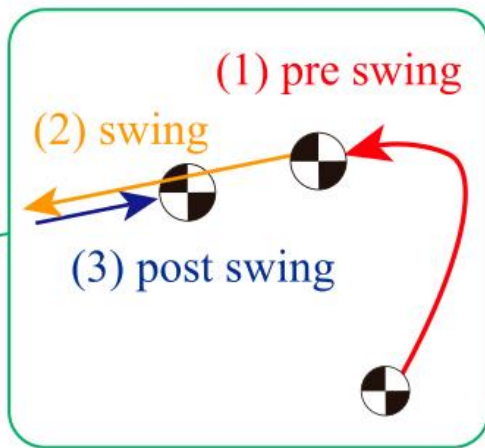
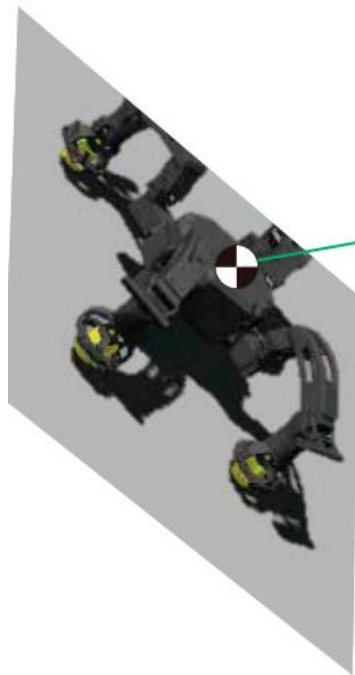
- Phase based CoM trajectory parameterization

$$\begin{bmatrix} m\mathbf{p}_G \times (\ddot{\mathbf{p}}_G - \mathbf{g}) + \dot{\mathbf{L}} \\ m(\ddot{\mathbf{p}}_G - \mathbf{g}) \end{bmatrix} = \begin{bmatrix} \mathcal{P}_c \\ \mathcal{R}_c \end{bmatrix} \mathbf{f}_c + \begin{bmatrix} \mathcal{P}_m \\ \mathcal{R}_m \end{bmatrix} \mathbf{f}_m$$

$$\mathbf{D}\mathbf{f}_c \geq 0,$$

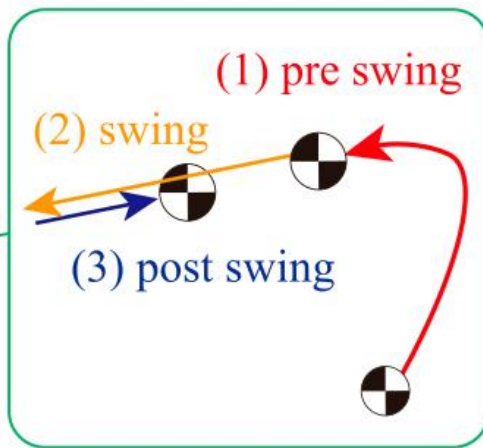
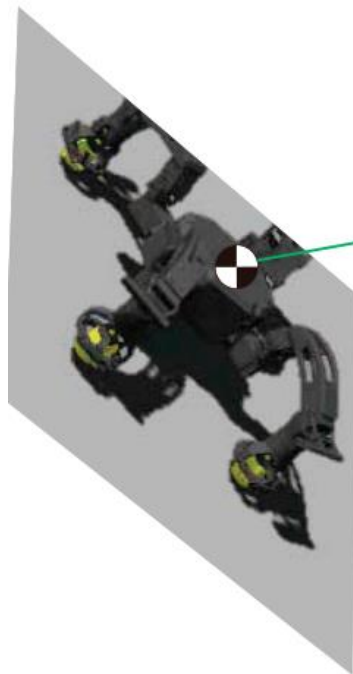


Find  $\mathbf{p}_G(\mathbf{t}), \mathbf{f}_c(\mathbf{t})$   
satisfying equations  
and inequality constraints!



# Multi-contact CoM trajectory generation for climbing

- Phase based CoM trajectory parameterization



<https://www.youtube.com/watch?v=6Ov8QqA7TNU>



# Multi-contact CoM trajectory generation for climbing

- Phase based CoM trajectory parameterization

$$\begin{bmatrix} m\mathbf{p}_G \times (\ddot{\mathbf{p}}_G - \mathbf{g}) + \dot{\mathbf{L}} \\ m(\ddot{\mathbf{p}}_G - \mathbf{g}) \end{bmatrix} = \begin{bmatrix} \mathcal{P}_c \\ \mathcal{R}_c \end{bmatrix} \mathbf{f}_c + \begin{bmatrix} \mathcal{P}_m \\ \mathcal{R}_m \end{bmatrix} \mathbf{f}_m$$

$$\mathbf{D}\mathbf{f}_c \geq 0,$$

Assumption 1

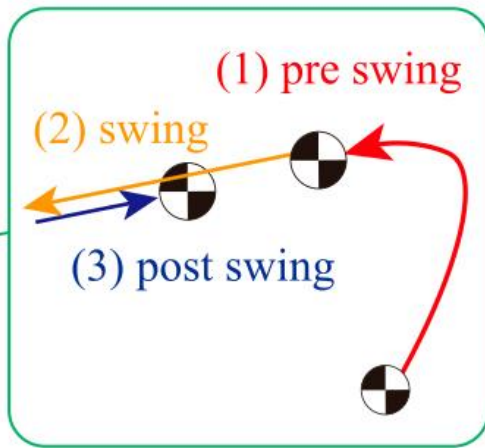
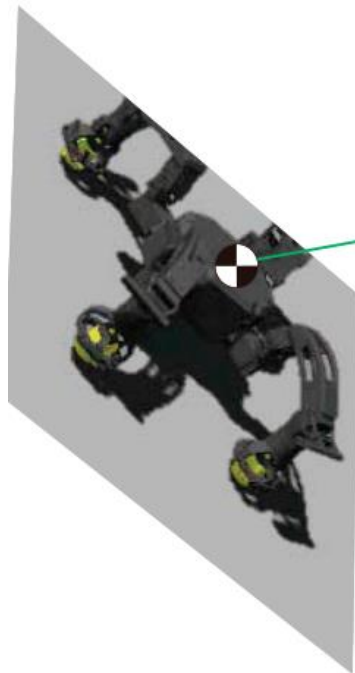
$\mathbf{p}_G \sim$  Hermite Cubic Spline

Assumption 2

Pre-swing motion is safe

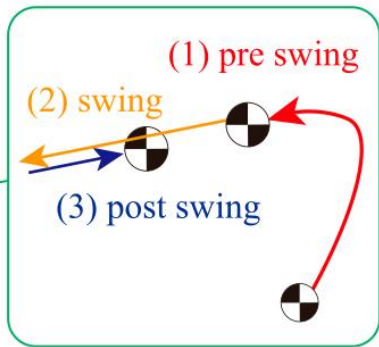
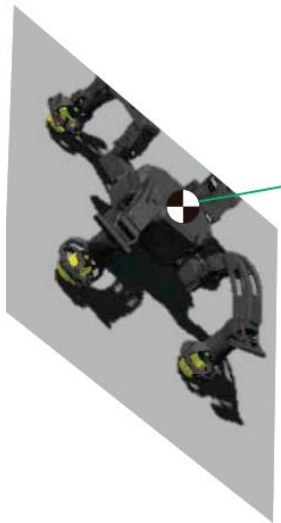
Assumption 3  $\mathbf{p}_G \times \ddot{\mathbf{p}}_G \rightarrow$  non-linear

$$\dot{\mathbf{p}}_G^{\text{swing}} \parallel \dot{\mathbf{p}}_G^{\text{post swing}}$$



# Multi-contact CoM trajectory generation for climbing

- Phase based CoM trajectory parameterization



- Cubic Hermite Spline

$$p_{HS}(t; T, \mathbf{p}_i, \mathbf{p}_g, \mathbf{v}_i, \mathbf{v}_g)$$

$$\Rightarrow \ddot{\mathbf{p}}_G^{\text{swing}}(t) = \alpha \mathbf{d}, \ddot{\mathbf{p}}_G^{\text{post swing}}(t) = \beta \mathbf{d},$$

(1) Pre swing    (2) Swing    (3) Post swing

$T$ (duration)	$T_1$	$T_2$	$T_3$
$\mathbf{p}_i$ (init position)	$\mathbf{p}_a$	$\mathbf{p}_1 = \mathbf{p}_b + (\frac{1}{2}\beta T_3^2 + \beta T_3 T_2 + \frac{1}{2}\alpha T_2^2)\mathbf{d}$	$\mathbf{p}_2$
$\mathbf{p}_g$ (goal position)	$\mathbf{p}_1$	$\mathbf{p}_2 = \mathbf{p}_b + \frac{1}{2}\beta T_3^2 \mathbf{d}$	$\mathbf{p}_b$
$\mathbf{v}_i$ (init velocity)	$\mathbf{0}$	$\mathbf{v}_1 = (-\beta T_3 - \alpha T_2)\mathbf{d}$	$\mathbf{v}_2$
$\mathbf{v}_g$ (goal velocity)	$\mathbf{v}_1$	$\mathbf{v}_2 = (-\beta T_3)\mathbf{d}$	$\mathbf{0}$

# Multi-contact CoM trajectory generation for climbing

- Phase based CoM trajectory parameterization

Assumption 2

pre-swing motion is safe

2) Swing

$$\begin{bmatrix} \mathcal{P}_s - [\mathbf{p}_b]_{\times} \mathcal{R}_s \\ \mathcal{R}_s \end{bmatrix} \mathbf{f}_c = \begin{bmatrix} \frac{1}{2} m [\mathbf{g}]_{\times} (T_2 - t)^2 \\ m \mathbf{I}_3 \end{bmatrix} \alpha \mathbf{d} + \begin{bmatrix} \frac{1}{2} m [\mathbf{g}]_{\times} T_3 (T_3 + 2T_2 - t) \\ 0 \end{bmatrix} \beta \mathbf{d} + \begin{bmatrix} \dot{\mathbf{L}} - (\mathcal{P}_s - [\mathbf{p}_b]_{\times} \mathcal{R}_s) \mathbf{f}_m \\ -m\mathbf{g} - \mathcal{R}_s \mathbf{f}_m \end{bmatrix}$$

$$\Leftrightarrow \mathbf{A}_s \mathbf{f}_c = \mathbf{B}_{sa}(t) \alpha \mathbf{d} + \mathbf{B}_{sb}(t) \beta \mathbf{d} + \mathbf{c}_s, \quad \forall t \in (0, T_2)$$

$$\mathbf{D}_s \mathbf{f}_c \geq 0$$

3) Post Swing

$$\begin{bmatrix} \mathcal{P}_f - [\mathbf{p}_b]_{\times} \mathcal{R}_f \\ \mathcal{R}_f \end{bmatrix} \mathbf{f}_c = \begin{bmatrix} \frac{1}{2} m [\mathbf{g}]_{\times} (T_3 - t)^2 \\ m \mathbf{I}_3 \end{bmatrix} \beta \mathbf{d} + \begin{bmatrix} \dot{\mathbf{L}} - (\mathcal{P}_f - [\mathbf{p}_b]_{\times} \mathcal{R}_f) \mathbf{f}_m \\ -m\mathbf{g} - \mathcal{R}_f \mathbf{f}_m \end{bmatrix}$$

$$\Leftrightarrow \mathbf{A}_f \mathbf{f}_c = \mathbf{B}_f(t) \beta \mathbf{d} + \mathbf{c}_f, \quad \forall t \in (0, T_3),$$

$$\mathbf{D}_f \mathbf{f}_c \geq 0$$

# Multi-contact CoM trajectory generation for climbing

- Problem solution for parameterized CoM trajectory generation

find  $\alpha$ ,  $\beta$ ,  $\mathbf{d}$  subject to

2) Swing

$$\mathbf{A}_s \mathbf{f}_c = \mathbf{B}_{sa}(t) \alpha \mathbf{d} + \mathbf{B}_{sb}(t) \beta \mathbf{d} + \mathbf{c}_s, \quad \forall t \in (0, T_2)$$

$$\mathbf{D}_s \mathbf{f}_c \geq 0$$

3) Post Swing

$$\mathbf{A}_f \mathbf{f}_c = \mathbf{B}_f(t) \beta \mathbf{d} + \mathbf{c}_f, \quad \forall t \in (0, T_3)$$

$$\mathbf{D}_f \mathbf{f}_c \geq 0$$



Still nonlinear terms: multiplication of variables,  $\alpha \mathbf{d}$ ,  $\beta \mathbf{d}$

# Multi-contact CoM trajectory generation for climbing

- Problem solution for parameterized CoM trajectory generation

## Strategy 1

Choose  $\gamma = \frac{\alpha}{\beta}$  that minimizes the max dist b/w CoM goal position & swing phase trajectory

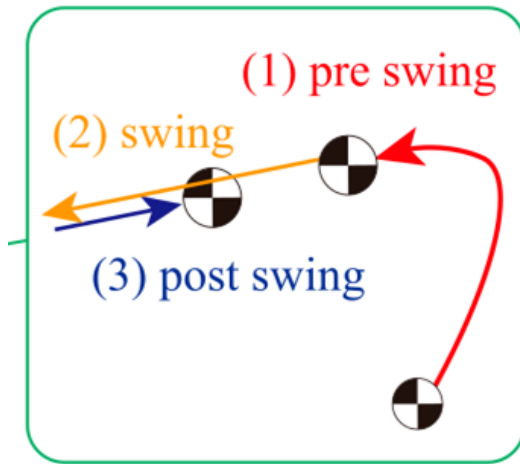
$$\gamma^* = \arg \min_{\gamma} \left( \max_t |g_{\gamma}(t)|, \quad t \in (0, T_2) \right),$$

where  $\mathbf{p}_s(t) - \mathbf{p}_b = g_{\gamma}(t) \cdot \beta \mathbf{d}, \quad t \in (0, T_2),$

$$g_{\gamma}(t) = \frac{1}{2} \gamma (T_2 - t)^2 + T_3 (T_2 - t) + \frac{1}{2} T_3^2,$$

Assuming  $\gamma^* \neq 0$  and solving for  $\gamma < 0$ , we get the following:

$$\gamma^* = -\frac{T_3 (T_2 + T_3) + T_3 \sqrt{(T_2 + T_3)^2 + T_2^2}}{T_2^2}. \quad (11)$$





# Multi-contact CoM trajectory generation for climbing

- Problem solution for parameterized CoM trajectory generation

Strategy 1

Choose  $\gamma = \frac{\alpha}{\beta}$  that minimizes the max dist b/w CoM goal position & swing phase trajectory.

find  ~~$\alpha, \beta, \mathbf{d}$~~   $\beta \mathbf{d}$  subject to

2) Swing

$$\mathbf{A}_s \mathbf{f}_c = (\mathbf{B}_{sa}(t) \gamma^* + \mathbf{B}_{sb}(t)) \beta \mathbf{d} + \mathbf{c}_s, \quad \forall t \in (0, T_2)$$

$$\mathbf{D}_s \mathbf{f}_c \geq 0$$

3) Post Swing

$$\mathbf{A}_f \mathbf{f}_c = \mathbf{B}_f(t) \beta \mathbf{d} + \mathbf{c}_f, \quad \forall t \in (0, T_3)$$

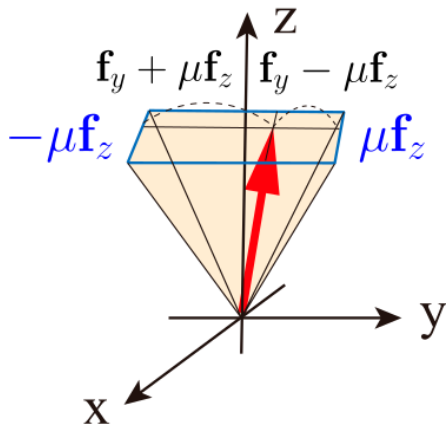
$$\mathbf{D}_f \mathbf{f}_c \geq 0$$

# Multi-contact CoM trajectory generation for climbing

- Problem solution for parameterized CoM trajectory generation

Strategy 2

Take  $\mathbf{f}_c$  that is located as close to the center of the cone as possible.



$$\begin{aligned} \min \quad & \mathbf{f}_c^\top \mathbf{W} \mathbf{f}_c = \sum_{c \in \mathcal{C}} \left( \frac{(f_x^c)^2 + (f_y^c)^2 + 2(\tilde{\mu}^c f_z^c)^2}{(\mu^c)^2} \right) \\ \text{s.t.} \quad & \mathbf{A} \mathbf{f}_c = \mathbf{b} \end{aligned}$$

$$\Rightarrow \mathbf{f}_c = \mathbf{W}^{-1} \mathbf{A}^\top (\mathbf{A} \mathbf{W}^{-1} \mathbf{A}^\top)^{-1} \mathbf{b}$$

$$\Rightarrow \text{Find } \mathbf{x} = \beta \mathbf{d}$$

$$\text{s.t. } \mathbf{D}_s \mathbf{A}_s^{-1} (\mathbf{B}_{sa}(t) \gamma^* + \mathbf{B}_{sb}(t)) \mathbf{x} + \mathbf{D}_s \mathbf{A}_s^{-1} \mathbf{c}_s \geq 0, \quad \forall t \in (0, T_2)$$

$$\mathbf{D}_f \mathbf{A}_f^{-1} \mathbf{B}_f(t) \mathbf{x} + \mathbf{D}_f \mathbf{A}_f^{-1} \mathbf{c}_f \geq 0, \quad \forall t \in (0, T_3).$$

# Multi-contact CoM trajectory generation for climbing

- Problem solution for parameterized CoM trajectory generation

## Strategy 3

Necessary and sufficient condition to satisfy inequality over the given time horizon.

$$\text{Find } \mathbf{x} = \beta \mathbf{d} \quad \text{s.t.} \quad \begin{aligned} & \left( \frac{1}{2} m f_s(t) \mathbf{D}_s \mathbf{A}_s^{-1}{}_{1;3} [\mathbf{g}]_{\times} + m \gamma^* \mathbf{D}_s \mathbf{A}_s^{-1}{}_{4;6} \right) \mathbf{x} + \mathbf{D}_s \mathbf{A}_s^{-1} \mathbf{c}_s \geq 0 \quad t \in (0, T_2) \\ & \left( \frac{1}{2} m f_f(t) \mathbf{D}_f \mathbf{A}_f^{-1}{}_{1;3} [\mathbf{g}]_{\times} + m \gamma^* \mathbf{D}_f \mathbf{A}_f^{-1}{}_{4;6} \right) \mathbf{x} + \mathbf{D}_f \mathbf{A}_f^{-1} \mathbf{c}_f \geq 0 \quad t \in (0, T_3) \end{aligned}$$

Proposition 1. Given an inequality  $(f(t) \mathbf{B}_1 + \mathbf{B}_2) \mathbf{x} + \mathbf{c} \geq 0$  defined over a bounded function,

$f_{\min} \leq f(t) \leq f_{\max}, \forall t \in (0, T)$ , if an inequality holds at the boundary values

$$(f_{\min} \mathbf{B}_1 + \mathbf{B}_2) \mathbf{x} + \mathbf{c} \geq 0, (f_{\max} \mathbf{B}_1 + \mathbf{B}_2) \mathbf{x} + \mathbf{c} \geq 0 \quad \text{for } \mathbf{x},$$

Then  $(f(t) \mathbf{B}_1 + \mathbf{B}_2) \mathbf{x} + \mathbf{c} \geq 0, \forall t \in (0, T)$

# Multi-contact CoM trajectory generation for climbing

- Problem solution for parameterized CoM trajectory generation

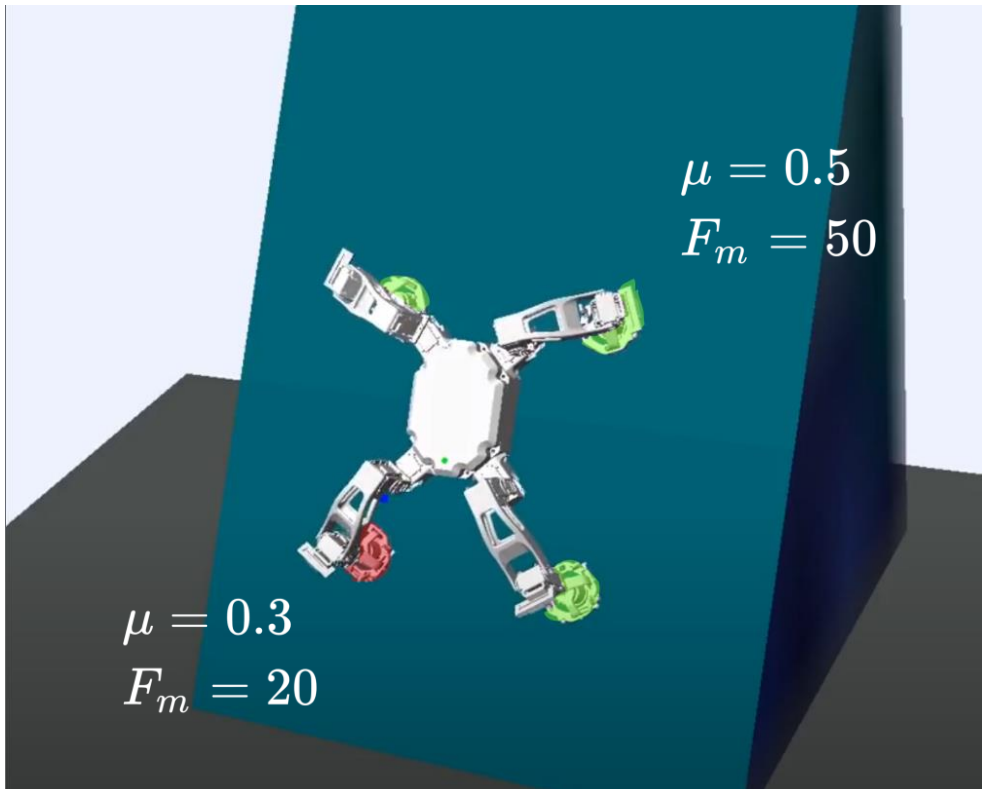
Strategy 3

Necessary and sufficient condition to satisfy inequality over the given time horizon.

$$\begin{aligned} & \Rightarrow \min \quad \|\mathbf{x}\|^2, \\ & \text{subject to} \quad \underbrace{\begin{bmatrix} \frac{1}{2}mf_{s,\min}\mathbf{D}_s\mathbf{A}_s^{-1}\mathbf{1}; 3[\mathbf{g}]_x + m\gamma^*\mathbf{D}_s\mathbf{A}_s^{-1}\mathbf{4}; 6 \\ \frac{1}{2}mf_{s,\max}\mathbf{D}_s\mathbf{A}_s^{-1}\mathbf{1}; 3[\mathbf{g}]_x + m\gamma^*\mathbf{D}_s\mathbf{A}_s^{-1}\mathbf{4}; 6 \\ \frac{1}{2}mf_{f,\min}\mathbf{D}_f\mathbf{A}_f^{-1}\mathbf{1}; 3[\mathbf{g}]_x + m\mathbf{D}_f\mathbf{A}_f^{-1}\mathbf{4}; 6 \\ \frac{1}{2}mf_{f,\max}\mathbf{D}_f\mathbf{A}_f^{-1}\mathbf{1}; 3[\mathbf{g}]_x + m\mathbf{D}_f\mathbf{A}_f^{-1}\mathbf{4}; 6 \end{bmatrix}}_{\mathbf{D}_x} \mathbf{x} + \underbrace{\begin{bmatrix} \mathbf{D}_s\mathbf{A}_s^{-1}\mathbf{c}_s \\ \mathbf{D}_s\mathbf{A}_s^{-1}\mathbf{c}_s \\ \mathbf{D}_f\mathbf{A}_f^{-1}\mathbf{c}_f \\ \mathbf{D}_f\mathbf{A}_f^{-1}\mathbf{c}_f \end{bmatrix}}_{\mathbf{d}_x} \geq 0 \end{aligned}$$

where  $\mathbf{x} = \beta\mathbf{d} \in \mathbb{R}^3$ ,  $\mathbf{D}_x \in \mathbb{R}^{70 \times 3}$ , and  $\mathbf{d}_x \in \mathbb{R}^{70}$

# Multi-contact CoM trajectory generation for climbing

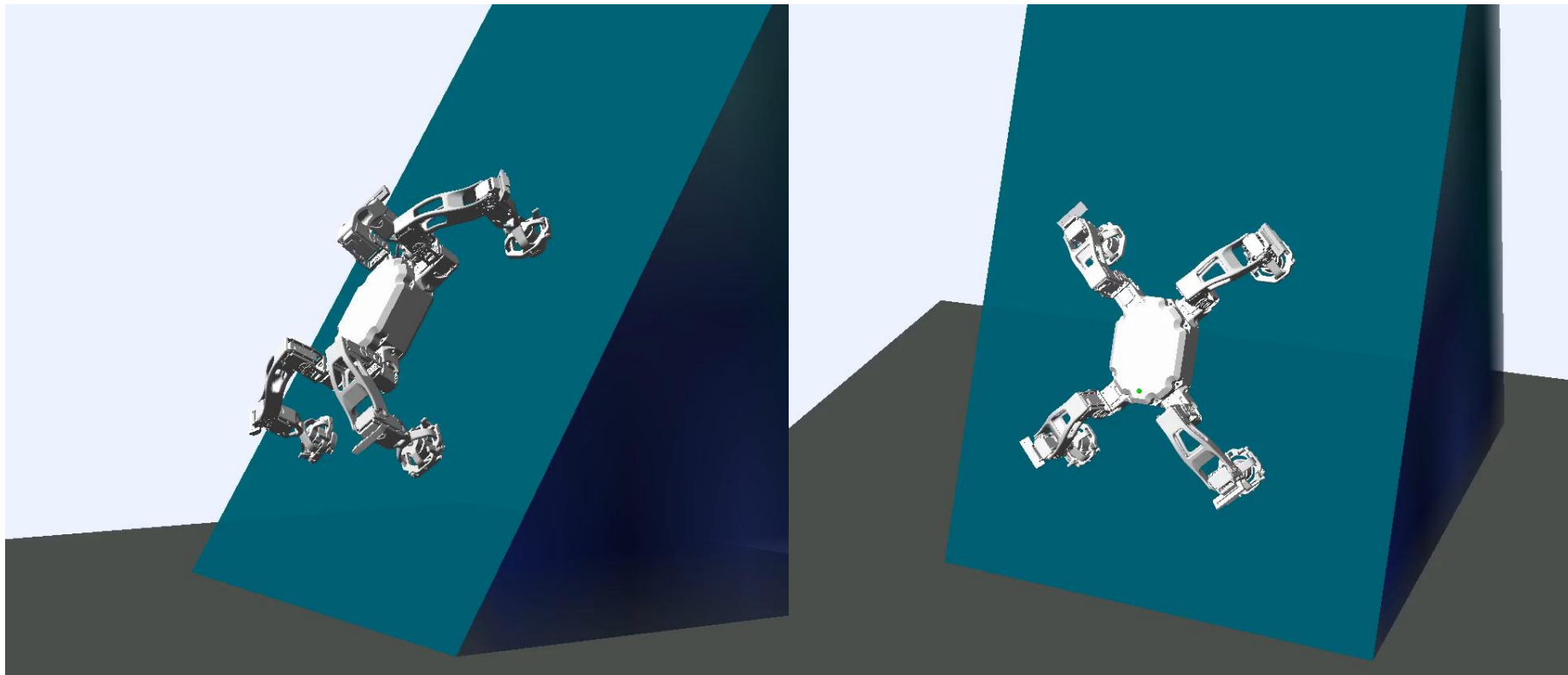


Scenario:

climbing on a flat slope  
 with weak adhesion  
 & slippery contact  
 at the left bottom foot

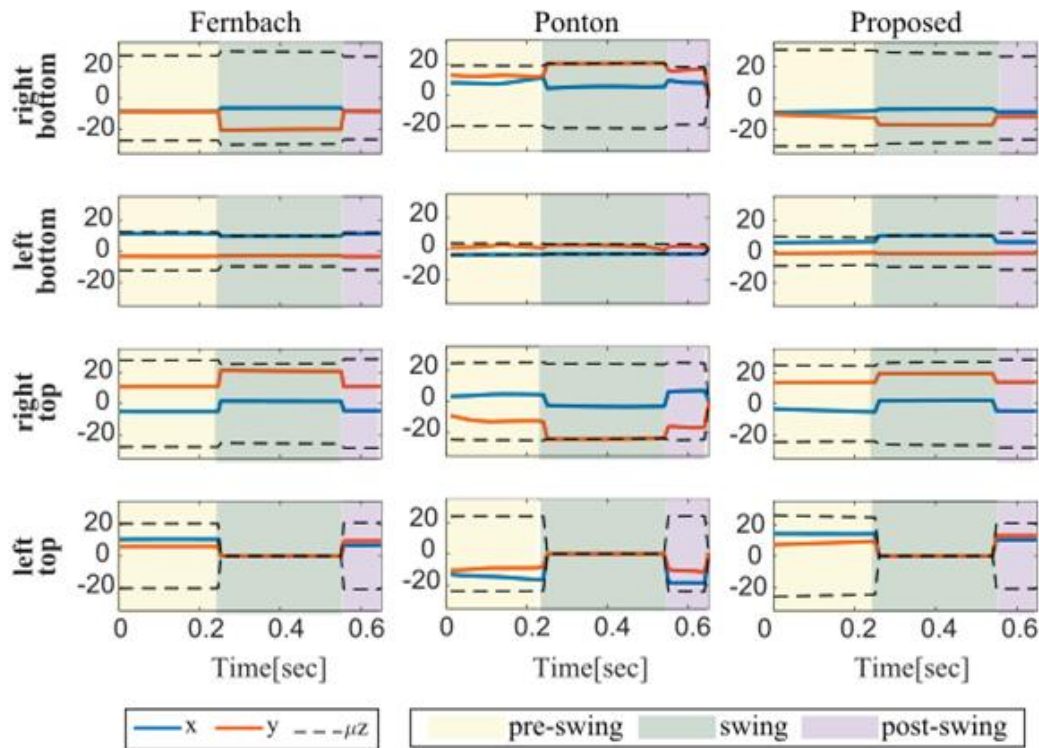
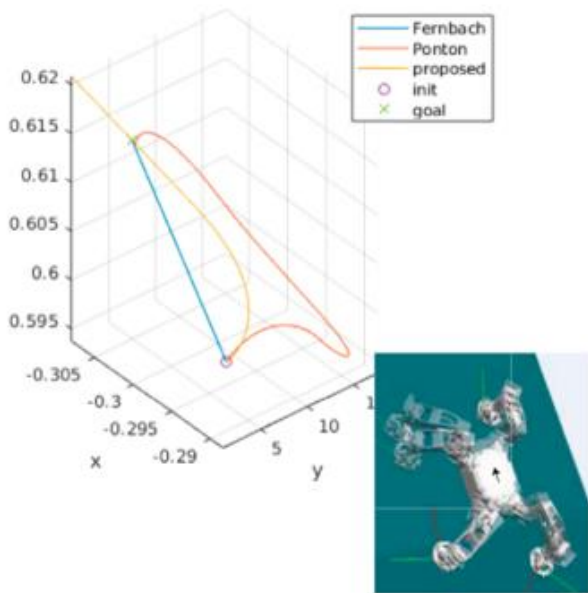


# Multi-contact CoM trajectory generation for climbing



# Comparison benchmark of CoM Trajectory Optimization

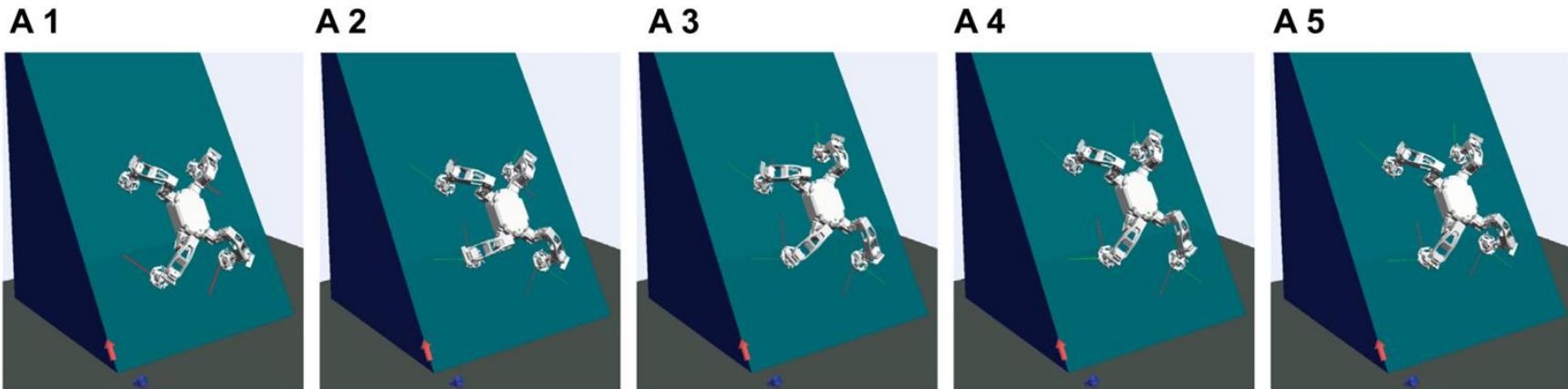
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\* Fernbach et al. "C-croc: Continuous and convex resolution of centroidal dynamic trajectories for legged robots in multicontact scenarios." TRO 2022

\* Ponton et al. "On time optimization of centroidal momentum dynamics." ICRA 2018

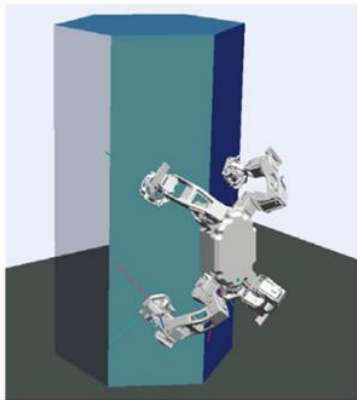
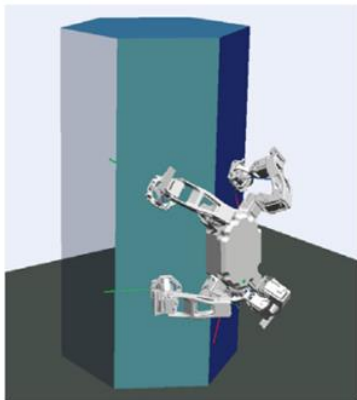
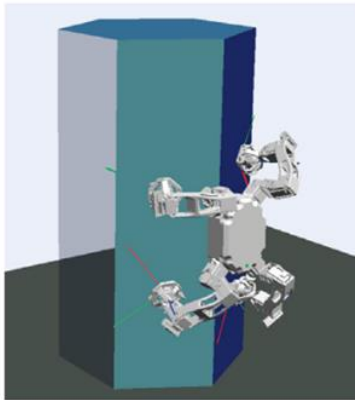
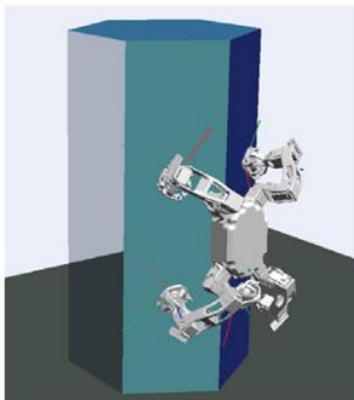
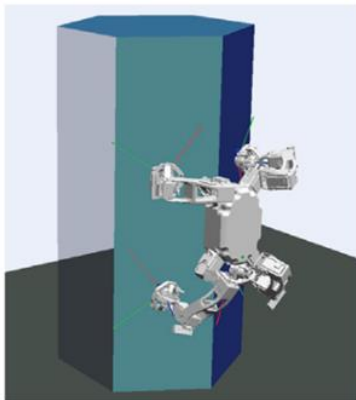
# Comparison benchmark of CoM Trajectory Optimization



Computation time [ms] for flat slope climbing scenario(motion horizon = 0.65 s)

Method	A 1		A 3		A 5	
	Swing foot	Proposed method	Left bottom	Right top	Left top	Right bottom
Proposed method			5.22e-02	5.23e-02	5.45e-02	5.67e-02
Ponton et al. (2018) ( $\Delta T = 10\text{ms}$ )			2,400	2,230	2,280	2,330
Ponton et al. (2018) ( $\Delta T = 50\text{ms}$ )			313	295	303	314
Fernbach et al. (2020)			7.51	7.32	7.81	7.35

# Comparison benchmark of CoM Trajectory Optimization

**B 1**

**B 2**

**B 3**

**B 4**

**B 5**


Computation time [ms] for hexagonal structure climbing scenario(motion horizon = 0.8s)

Method	Swing foot	Left bottom	Right top	Left top	Right bottom
	Proposed method	5.29e-02	5.78e-02	5.33e-02	5.91e-02
	Ponton et al. (2018) ( $\Delta T = 50\text{ms}$ )	365	332	326	379
	Fernbach et al. (2020)	1.40	1.58	1.40	1.59

## **(ii) Contact parameter estimation and CoM re-planning**

# Contact parameter estimation and CoM re-planning

## - Friction and magnetic adhesion force estimation

- Frictional force(tangential) equation:  $f_t = \sqrt{f_x^2 + f_y^2}$ ,

$$\hat{f}_t = \hat{\mu}(f_z + \hat{f}_m),$$

- Unknown environment parameters:  $\theta = [\hat{\mu}, \hat{\mu} \hat{f}_m]^\top$

### i) Least squares estimator

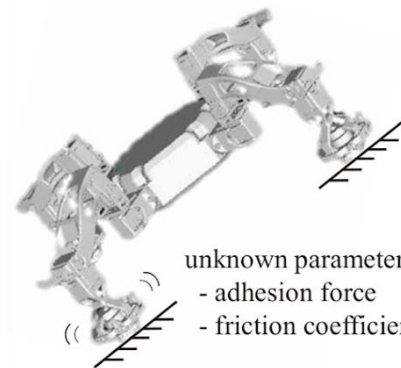
$$\begin{bmatrix} \sum f_z(t) & \sum 1 \\ \sum f_z(t)^2 & \sum f_z(t) \end{bmatrix} \begin{bmatrix} \hat{\mu} \\ \hat{\mu} \hat{f}_m \end{bmatrix} = \begin{bmatrix} \sum f_t(t) \\ \sum f_t(t) f_z(t) \end{bmatrix}$$

### ii) Kalman filter like estimator

$$\theta_{k+1} = \theta_k + w_k, \quad w_k \sim \mathcal{N}(0, Q_k),$$

$$z_k = H_k \theta_k + v_k, \quad v_k \sim \mathcal{N}(0, R_k),$$

where  $z_k = \begin{bmatrix} f_t(t_k - T + 1) \\ \vdots \\ f_t(t_k) \end{bmatrix}, \quad H_k = \begin{bmatrix} f_z(t_k - T + 1) & 1 \\ \vdots & \vdots \\ f_z(t_k) & 1 \end{bmatrix}$



### ➡ Kalman filter solution

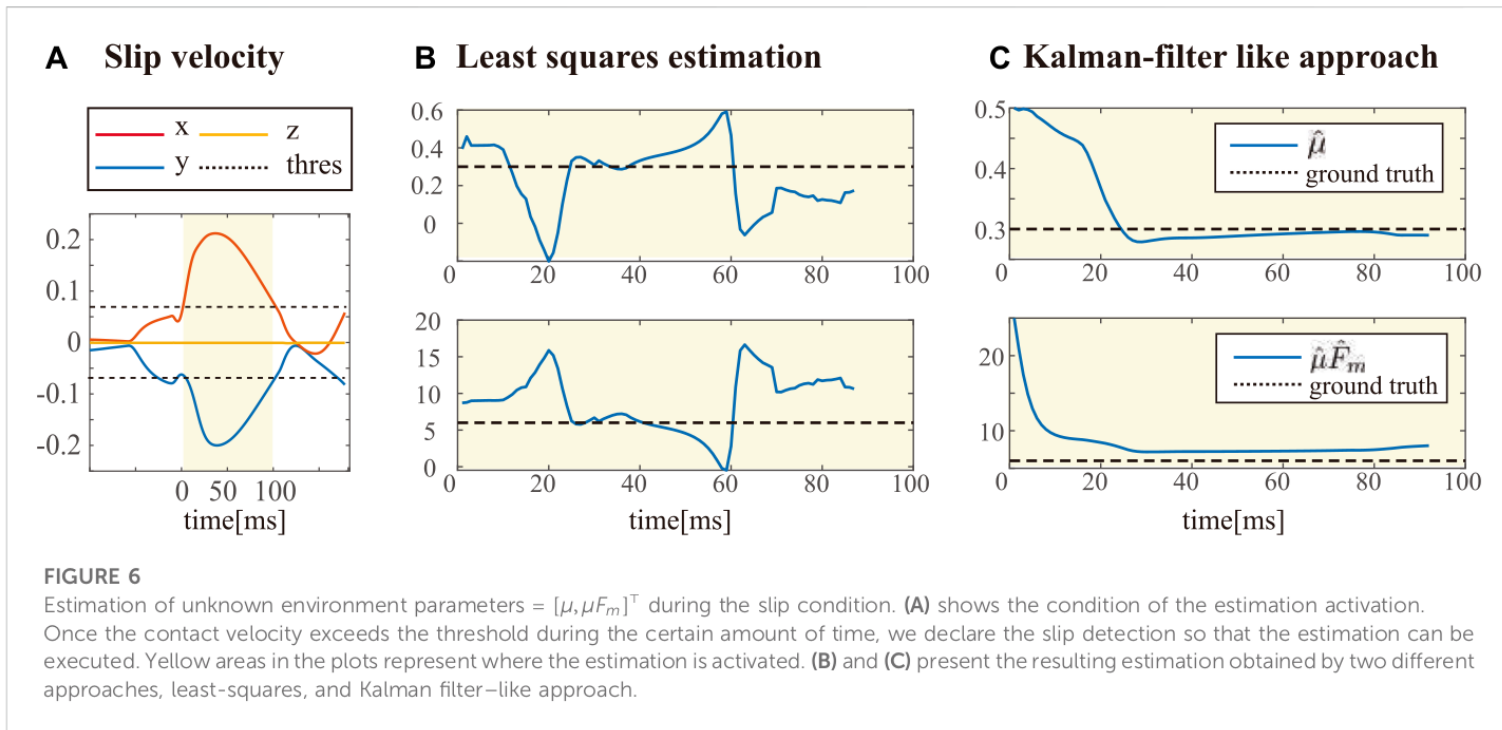
$$K_k = (P_{k-1} + Q_k) H_k^\top (H_k (P_{k-1} + Q_k) H_k^\top + R_k)^{-1}$$

$$P_k = (I - K_k H_k) (P_{k-1} + Q_k),$$

$$\hat{\theta}_k = \hat{\theta}_{k-1} + K_k (z_k - H_k \hat{\theta}_{k-1}).$$

# Contact parameter estimation and CoM re-planning

- Friction and magnetic adhesion force estimation





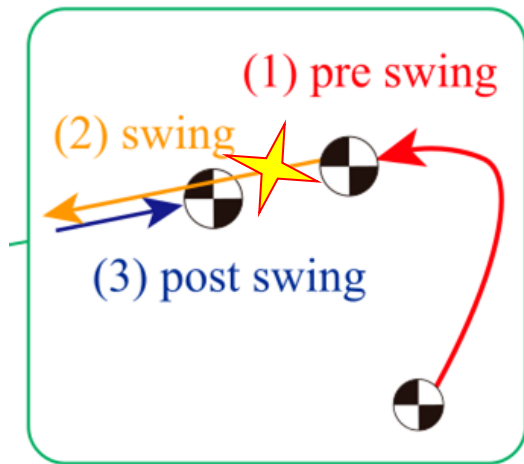
# Contact parameter estimation and CoM re-planning

- CoM re-planning for slip reflex

CoM replanning required during the swing phase?

(2) Swing

(3) Post swing



$T$  (duration)

$$T_2' = T_2 - t_{\text{now}}$$

$T_3$

$\mathbf{p}_i$  (init position)

$\mathbf{p}_a$

$\mathbf{p}_1$

$\mathbf{p}_g$  (goal position)

$$\mathbf{p}_1 = \mathbf{p}_a + \frac{1}{2} \alpha T_2'^2 \mathbf{d}$$

$\mathbf{p}_b$

$\mathbf{v}_i$  (init velocity)

$\mathbf{0}$

$\mathbf{v}_1$

$\mathbf{v}_g$  (goal velocity)

$$\mathbf{v}_1 = \alpha T_2' \mathbf{d}$$

$\mathbf{0}$

➡ Similarly, we can formulate the reduced problem

$$\begin{aligned} \min \quad & \|\mathbf{x}\|^2, \\ \text{subject to} \quad & \mathbf{D}_x' \mathbf{x} + \mathbf{d}_x' \geq 0 \end{aligned}$$

**(iii) Online weight adaptation to  
stabilize slippery motions**


# Online weight adaptation to stabilize slippery motions

$$\min_{\delta \ddot{\mathbf{q}}, \mathbf{F}_r, \ddot{\mathbf{x}}_c}, \delta \ddot{\mathbf{q}}^\top \mathbf{W}_{\ddot{\mathbf{q}}} \delta \ddot{\mathbf{q}} + \boxed{\mathbf{F}_r^\top \mathbf{W}_f \mathbf{F}_r} + \ddot{\mathbf{x}}_c^\top \mathbf{W}_c \ddot{\mathbf{x}}_c,$$

- Slippage rate:  $\alpha_s$  ( $>1$ , bigger if more slip)

$$\alpha_s = \begin{cases} 1 & \text{if } \|\mathbf{v}_{\text{contact foot}}\| \leq v_{\text{threshold}} \\ \frac{\|\mathbf{v}_{\text{contact foot}}\|}{v_{\text{threshold}}} & \text{otherwise} \end{cases}$$

$$\begin{bmatrix} fx \\ fy \\ fz \\ \vdots \end{bmatrix}^\top \begin{bmatrix} \omega_{fx} & 0 & 0 & \cdots \\ 0 & \omega_{fy} & 0 & \cdots \\ 0 & 0 & \omega_{fz} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} fx \\ fy \\ fz \\ \vdots \end{bmatrix}$$



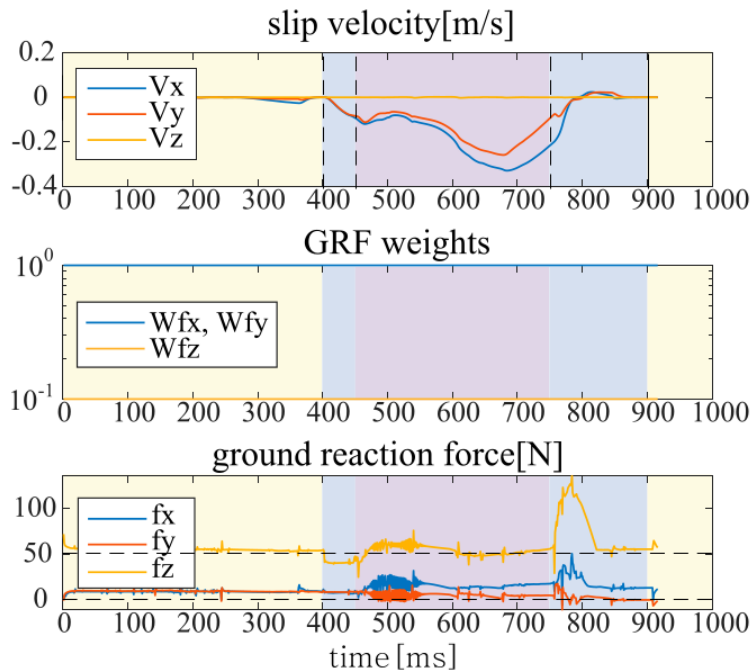
- Weight adaptation to the slippage

$$\Rightarrow w_{fx_i}, w_{fy_i} = \begin{cases} \alpha_{s_i} w_{xy}, & \text{if } \alpha_{s_i} > 1, \\ \frac{1}{\alpha_{s_i}} w_{xy}, & \text{otherwise.} \end{cases} \quad \text{and, } w_{fz_i} = \begin{cases} \frac{1}{\alpha_{s_i}} w_z, & \text{if } \alpha_{s_i} > 1, \\ \alpha_{s_i} w_z, & \text{otherwise.} \end{cases}$$

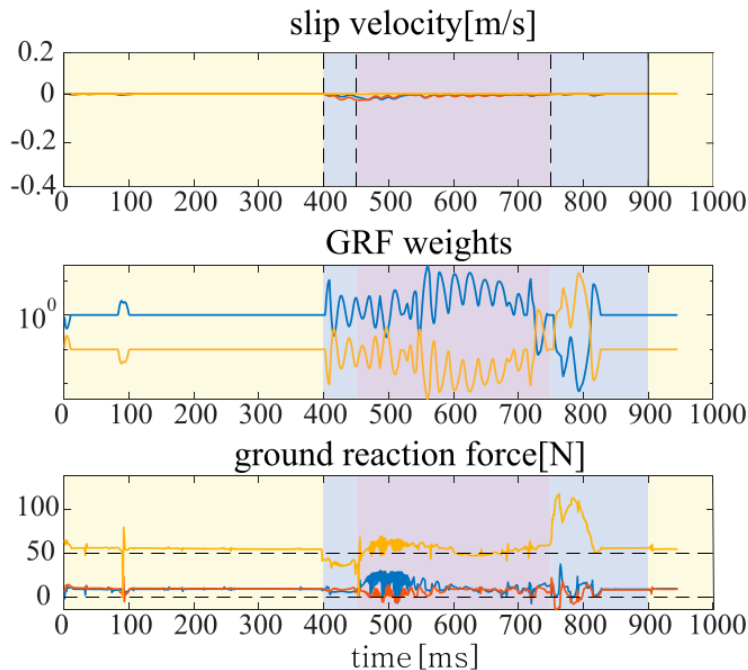


# Slippage reduction performance of weight adaptation

**A** w/o weight adaptation



**B** with weight adaptation



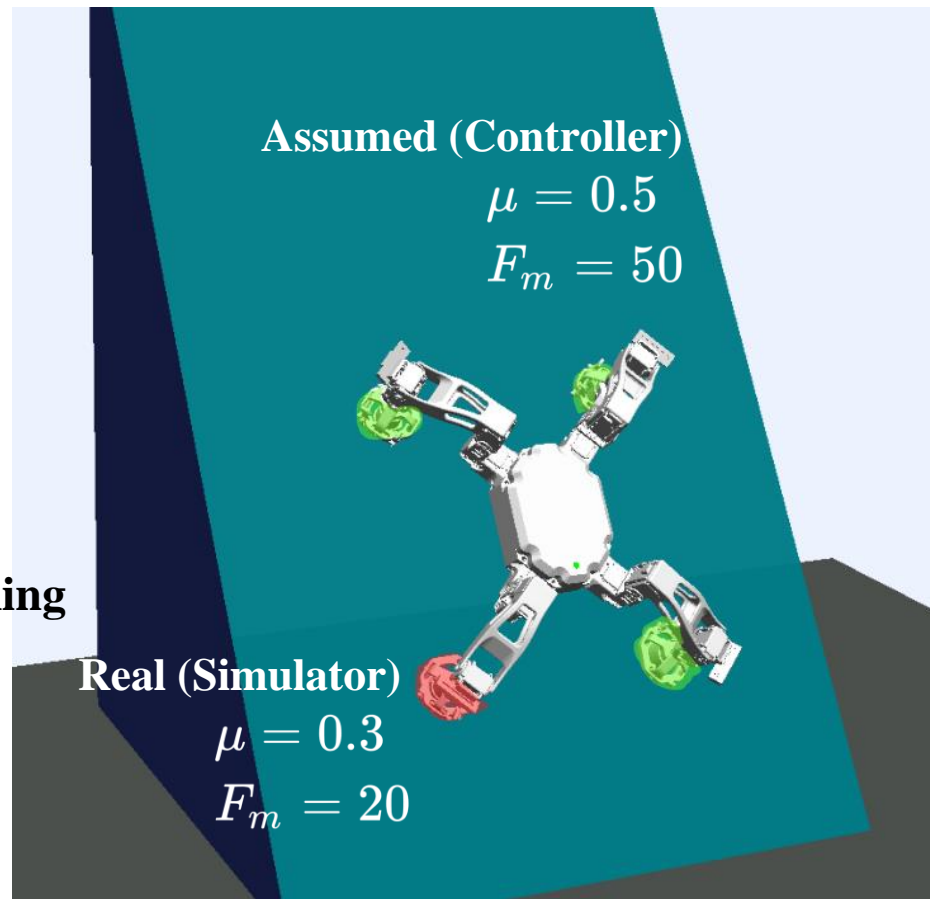
# Simulation Results

Climbing in unknown slippery conditions

## Scenario

Climbing on a flat slope with  
one foot in unknown slippery condition

- A. Without Any Adaptation**
- B. Parameter Estimation + CoM Replanning**
- C. Online Weight Adaptation**
- D. Both Strategies(B & C)**



1. W/O any adaptation

2. Param Estimation + CoM Replanning

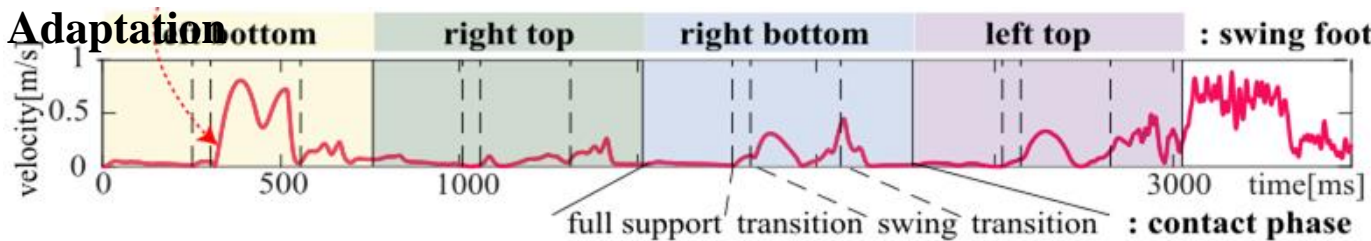
3. Weight Adaptation for QP-based WBC

4. Both (replanning+weight adaptation)

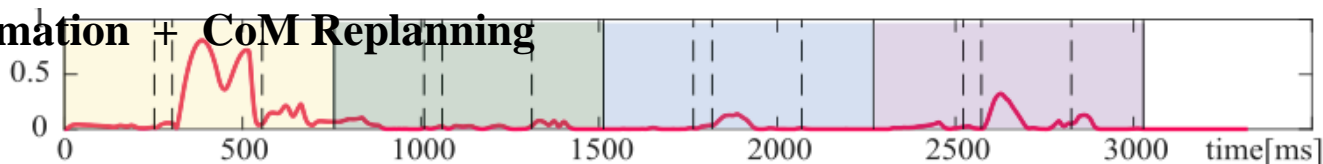


# Velocity of the left bottom foot ( in unknown slippery condition )

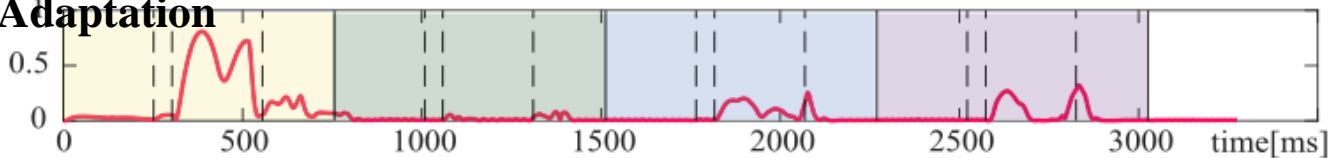
## A. Without Any Adaptation



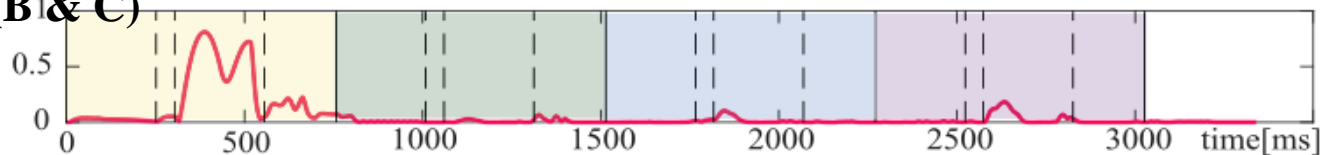
## A. Parameter Estimation + CoM Replanning



## A. Online Weight Adaptation



## A. Both Strategies(B & C)



# Thank you for listening!

## Q&A