A Constrained Iterative LQR Solver for the Trajectory Optimization Framework Horizon

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1. Contribution at a glance

- An ILQR algorithm for general **equality-constrained** problems
- **O(N) complexity** w.r.t. horizon length
- Computes a **linear policy** for both the control input $\delta \boldsymbol{u}$ and Lagrange multipliers $\delta \boldsymbol{\mu}$, $\delta \boldsymbol{\lambda}$
- Exploit Lagrange multipliers estimate to implement an **exact L1 line search** strategy
- Extensive **validation** on complex robotic examples

2. Problem definition

A **discrete-time** Trajectory Optimization (TO) problem, with **equality constraints**

$$\min_{x_{0:N},\, u_{0:N-1}} \sum_{k=0}^{N-1} \ell_k(x_k, u_k) + \ell_N(x_N)$$
$$\text{s.t.} \quad x_{k+1} = F(x_k, u_k)$$
$$h_k(x_k, u_k) = 0, \quad h_N(x_N) = 0$$

Note the final constraint cannot be deal with via projection/nullspace approaches!

3. Approach outline

Our strategy

- Apply **Newton's method** to the KKT conditions for the TO problem
- Solve the resulting linear system with **Riccati-like recursions** (backward pass + forward pass)

1) Hypotesize the following relation hold at node k

$$S_{k+1}\,\boldsymbol{\delta x}_{k+1} + V_{k+1}^T\,\boldsymbol{\delta \nu}_{k+1} - \delta\lambda_k = -s_{k+1}$$
$$V_{k+1}\,\boldsymbol{\delta x}_{k+1} = -v_{k+1}$$

show that it holds at $k-1$, too.

2) Back-propagate constraint via the dynamics

$$C_k\,\boldsymbol{\delta x}_k + \boldsymbol{D_k}\,\boldsymbol{\delta u}_k = c_k$$

3) Handle rank-deficiency of D_k. A generic state-only constraint cannot be solved by a single control input!

- Separate feasible-infeasible components at time k
- Do it also for Lagrangian multipliers

4. Globalization strategy

Promote convergence to a **local minimum** by enforcing the decrease of a **merit function**

$$m(X) = L(X) + \gamma\,\|H(X)\|_1,$$

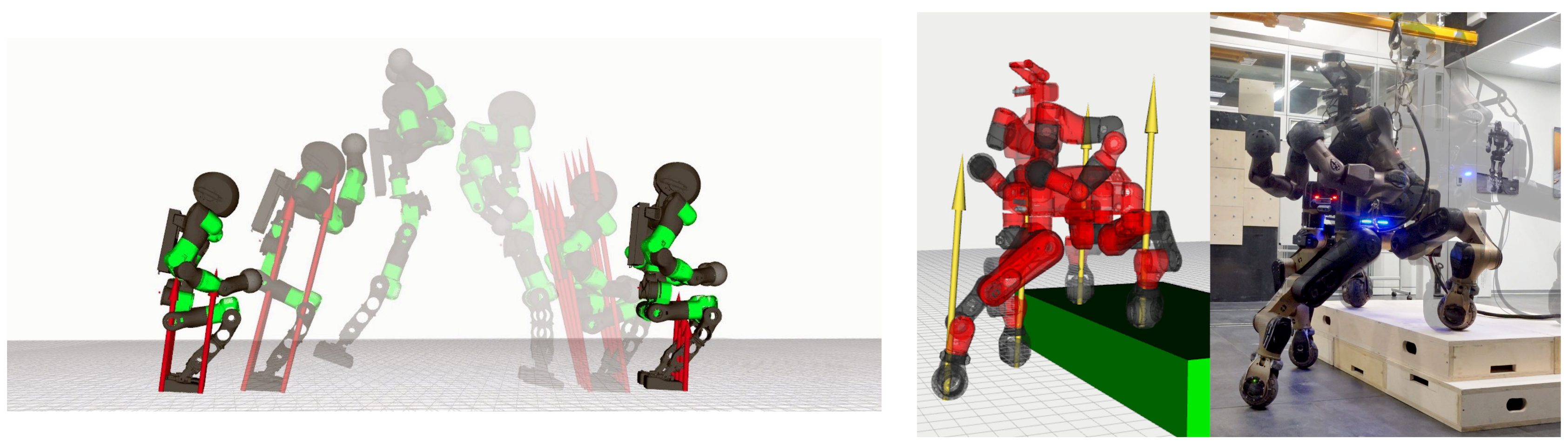
The merit function $m(X)$ is **exact**[a] if

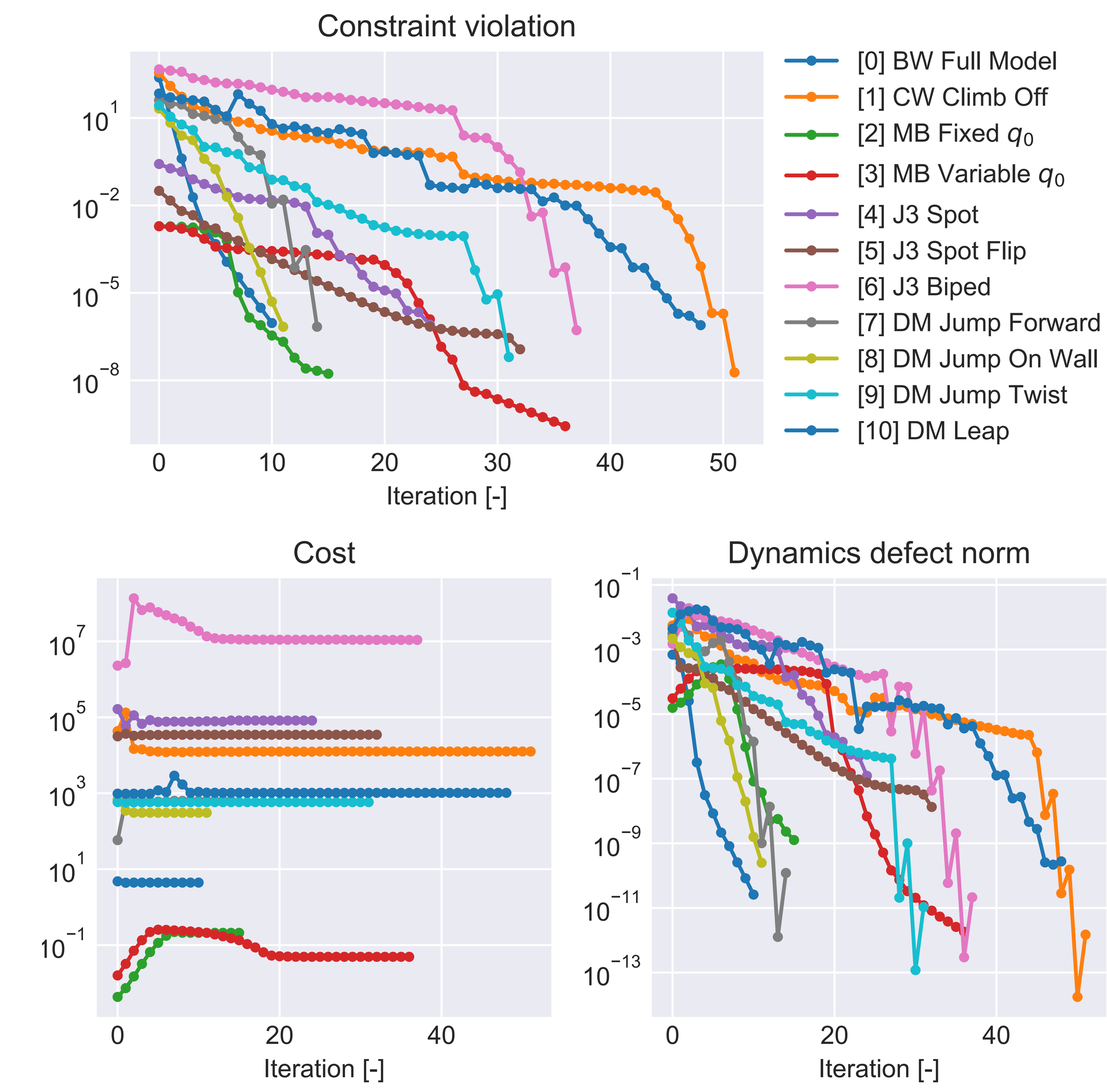
$$\gamma > \max\{\|\lambda^*_{0:N-1}\|_\infty,\ \|\mu^*_{0:N}\|_\infty\}$$

We can exploit the computed Lagrangian multiplier estimates to **tune γ automatically**

[a] A merit function is said to be *exact* if its local minima are also local minima for the original constrained problem.

5. Validation





- **Behaviors** entirely obtained via **constraints**
- Contact model, centroidal dynamics enforced via **constraints**
- Online (RTI) and offline
- 3[rd] order kino-dynamic model

Acknowledgements

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