

# Challenges for closed-loop nonlinear model predictive control on legged robots

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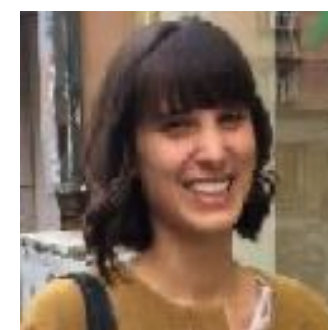


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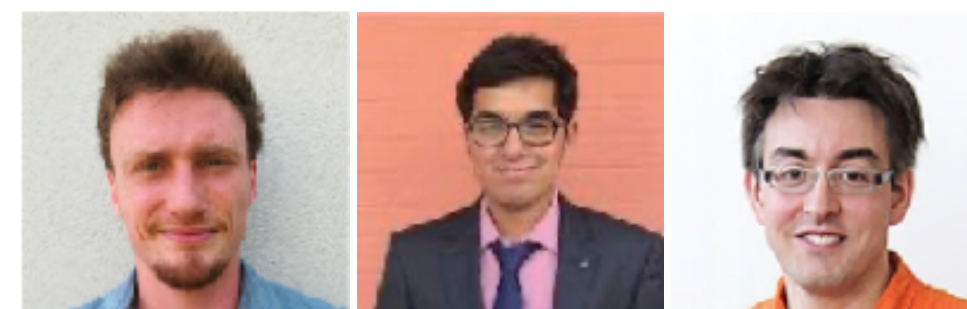
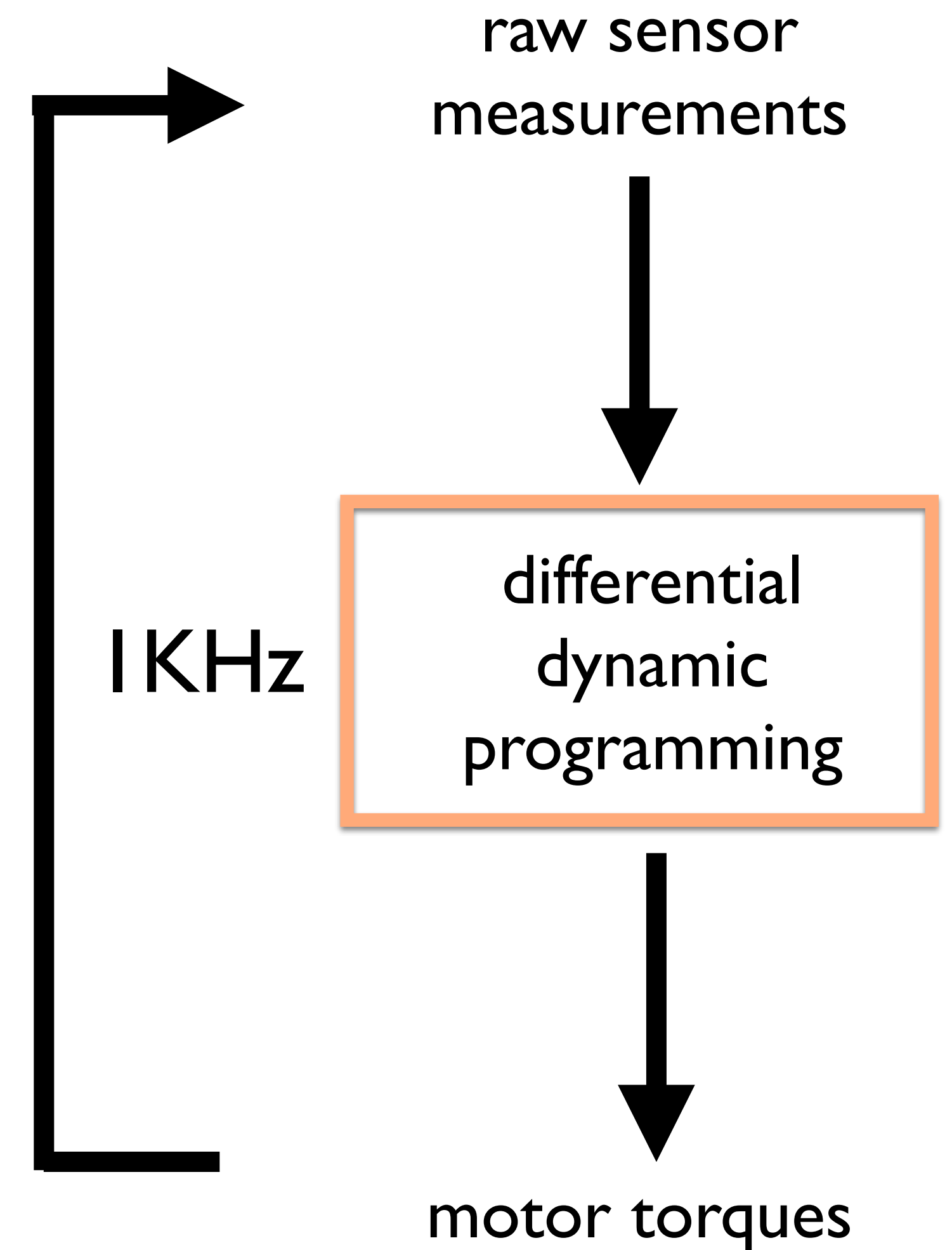
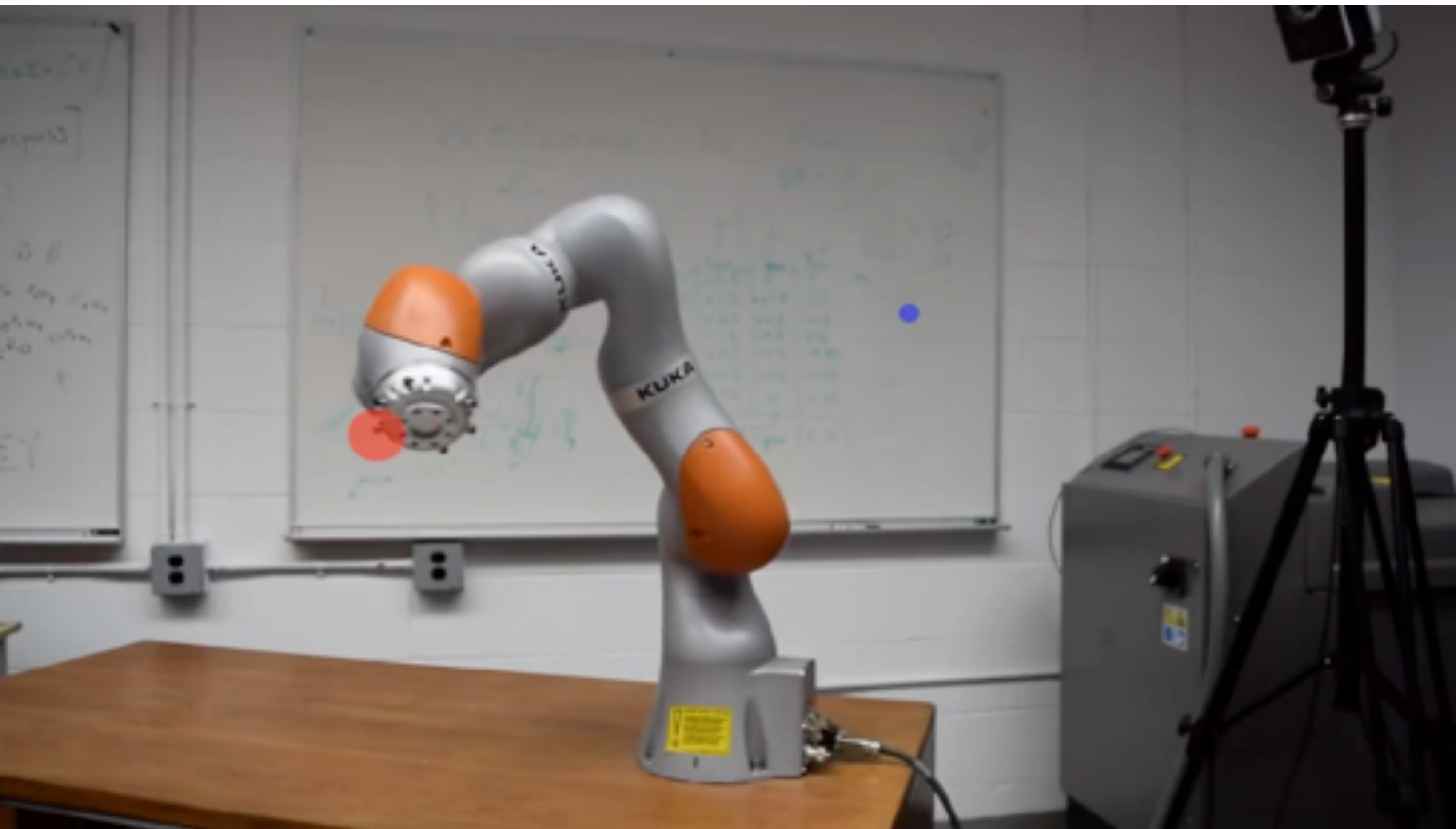
European  
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Horizon 2020  
European Union funding  
for Research & Innovation





# Closed-loop nonlinear model predictive control



[Kleff et al. ICRA 2021]

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{\tau}$$

# Nonlinear whole-body MPC for legged robots

unactuated dynamics = evolution of momentum

$$m\dot{\mathbf{r}} = \mathbf{h}_{\text{linear}}$$

$$\dot{\mathbf{h}} = \begin{bmatrix} M\mathbf{g} + \sum_e \mathbf{f}_e \end{bmatrix}$$



Decomposition of optimal control problem

equ I optimization of momentum + contact forces

II kinematics optimization

equivalent to a manipulator

any combination of motions and contact forces will satisfy actuated dynamics  
(ignoring actuation limits)



# centroidal dynamics optimization

$$\min_{\substack{\mathbf{F}_c, \mathbf{x}_{CoM} \\ \dot{\mathbf{x}}_{CoM}, \mathbf{L}_{CoM}}} \sum_t \phi_F(\mathbf{F}_c) + \phi_x(\mathbf{x}_{CoM}, \dot{\mathbf{x}}_{CoM}, \mathbf{L}_{CoM})$$

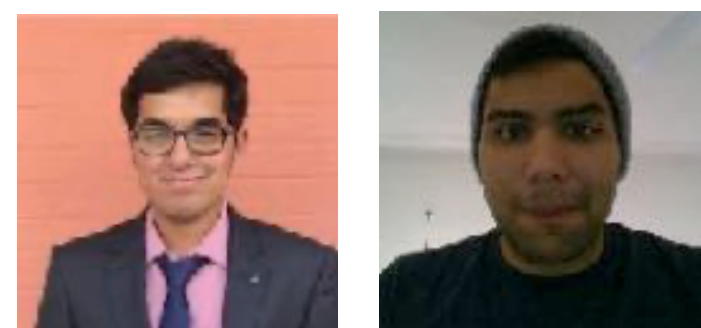
subject to

$$m\ddot{\mathbf{x}}_{CoM} = m\mathbf{g} + \sum_c \mathbf{F}_c$$

$$\dot{\mathbf{L}}_{CoM} = \sum_c (\mathbf{r}_c - \mathbf{x}_{CoM}) \times \mathbf{F}_c$$

$$\mathbf{F}_c \in \text{friction cones}$$

$$(\mathbf{x}_{CoM}, \dot{\mathbf{x}}_{CoM}, \mathbf{L}_{CoM}) \in \text{Box Constraints}$$



[Meduri et al., [arxiv.org/abs/2201.07601](https://arxiv.org/abs/2201.07601), T-RO in press]

# centroidal dynamics optimization

$$\min_{\substack{\mathbf{F}_c, \mathbf{x}_{CoM} \\ \dot{\mathbf{x}}_{CoM}, \mathbf{L}_{CoM}}} \sum_t \phi_F(\mathbf{F}_c) + \phi_x(\mathbf{x}_{CoM}, \dot{\mathbf{x}}_{CoM}, \mathbf{L}_{CoM})$$

Separable cost  
forces vs. positions/momentum

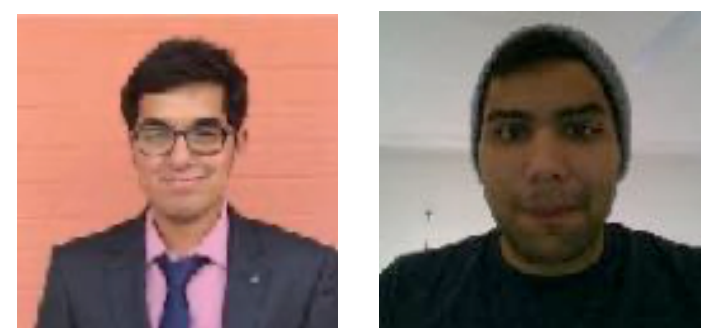
subject to

$$\begin{aligned} m\ddot{\mathbf{x}}_{CoM} &= m\mathbf{g} + \sum_c \mathbf{F}_c \\ \dot{\mathbf{L}}_{CoM} &= \sum_c (\mathbf{r}_c - \mathbf{x}_{CoM}) \times \mathbf{F}_c \end{aligned}$$

Equality constraints  
are bi-linear

$$\begin{aligned} \mathbf{F}_c &\in \text{friction cones} \\ (\mathbf{x}_{CoM}, \dot{\mathbf{x}}_{CoM}, \mathbf{L}_{CoM}) &\in \text{Box Constraints} \end{aligned}$$

Inequality constraints  
are easy to project  
(indicator functions)





$$\min_{\mathbf{F}_c, \mathbf{x}_{CoM}, \dot{\mathbf{x}}_{CoM}, \mathbf{L}_{CoM}} \sum_t \phi_F(\mathbf{F}_c) + \phi_x(\mathbf{x}_{CoM}, \dot{\mathbf{x}}_{CoM}, \mathbf{L}_{CoM})$$

subject to

$$m\ddot{\mathbf{x}}_{CoM} = m\mathbf{g} + \sum_c \mathbf{F}_c$$

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$\mathbf{F}_c \in \text{friction cones}$

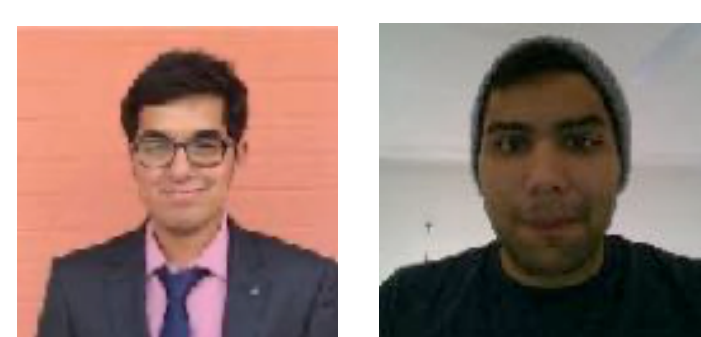
$(\mathbf{x}_{CoM}, \dot{\mathbf{x}}_{CoM}, \mathbf{L}_{CoM}) \in \text{Box Constraints}$

## ADMM for centroidal dynamics

- 1) solve **force problem** (positions are fixed)  
=> simple QP
- 2) solve **position problem** (forces are fixed)  
=> simple QP
- 3) Dual variables update  
(linear analytic formula)  
=> iterate until convergence  
(early termination for MPC)

## FISTA for QP solver (Fast Iterative Shrinkage Thresholding Algorithm)

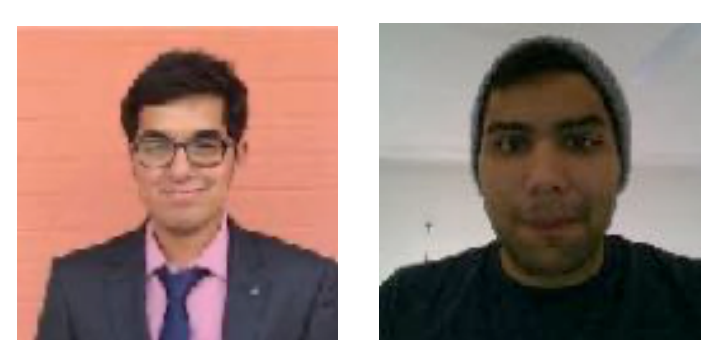
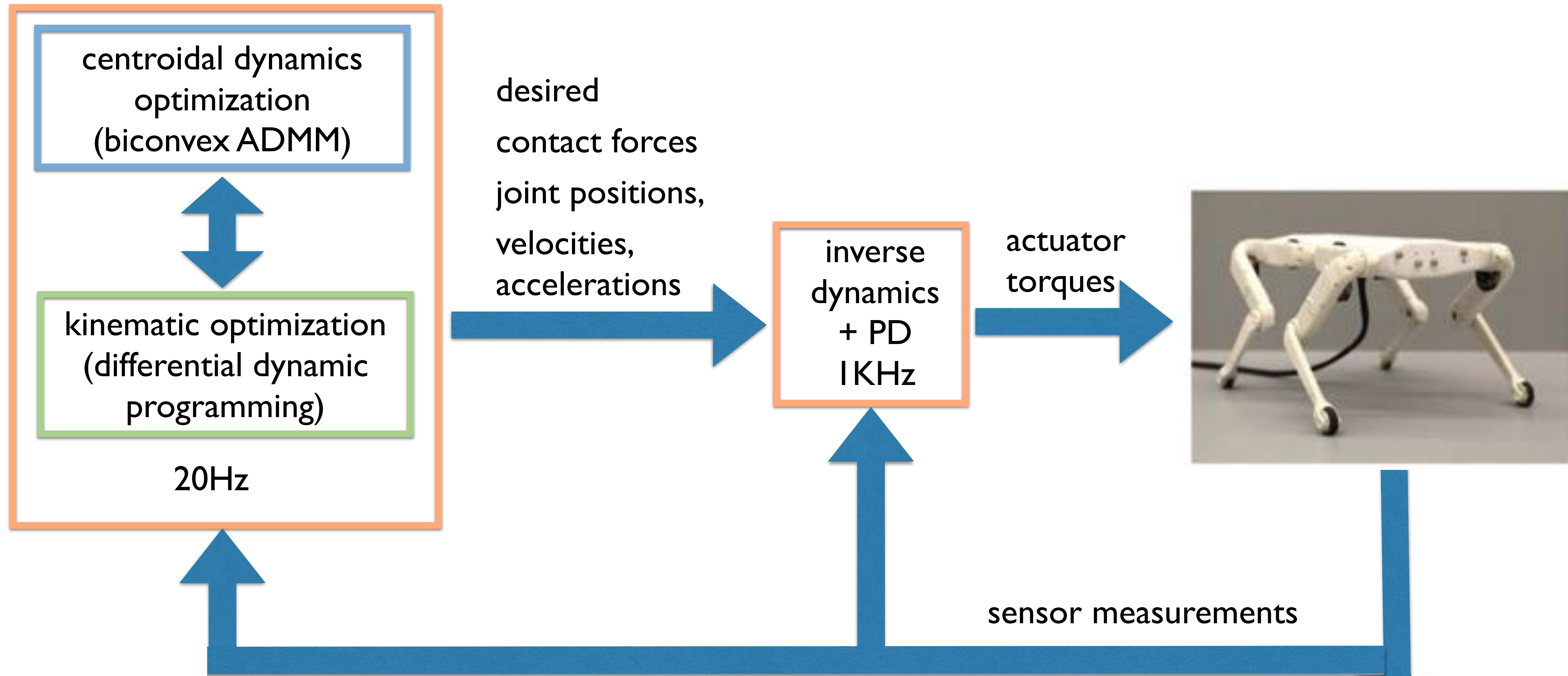
- Nesterov accelerated gradient descent + projection operators for constraints
- First order method (only gradients) with second order convergence guarantees
- Easy to implement



[Meduri et al., [arxiv.org/abs/2201.07601](https://arxiv.org/abs/2201.07601), T-RO in press]

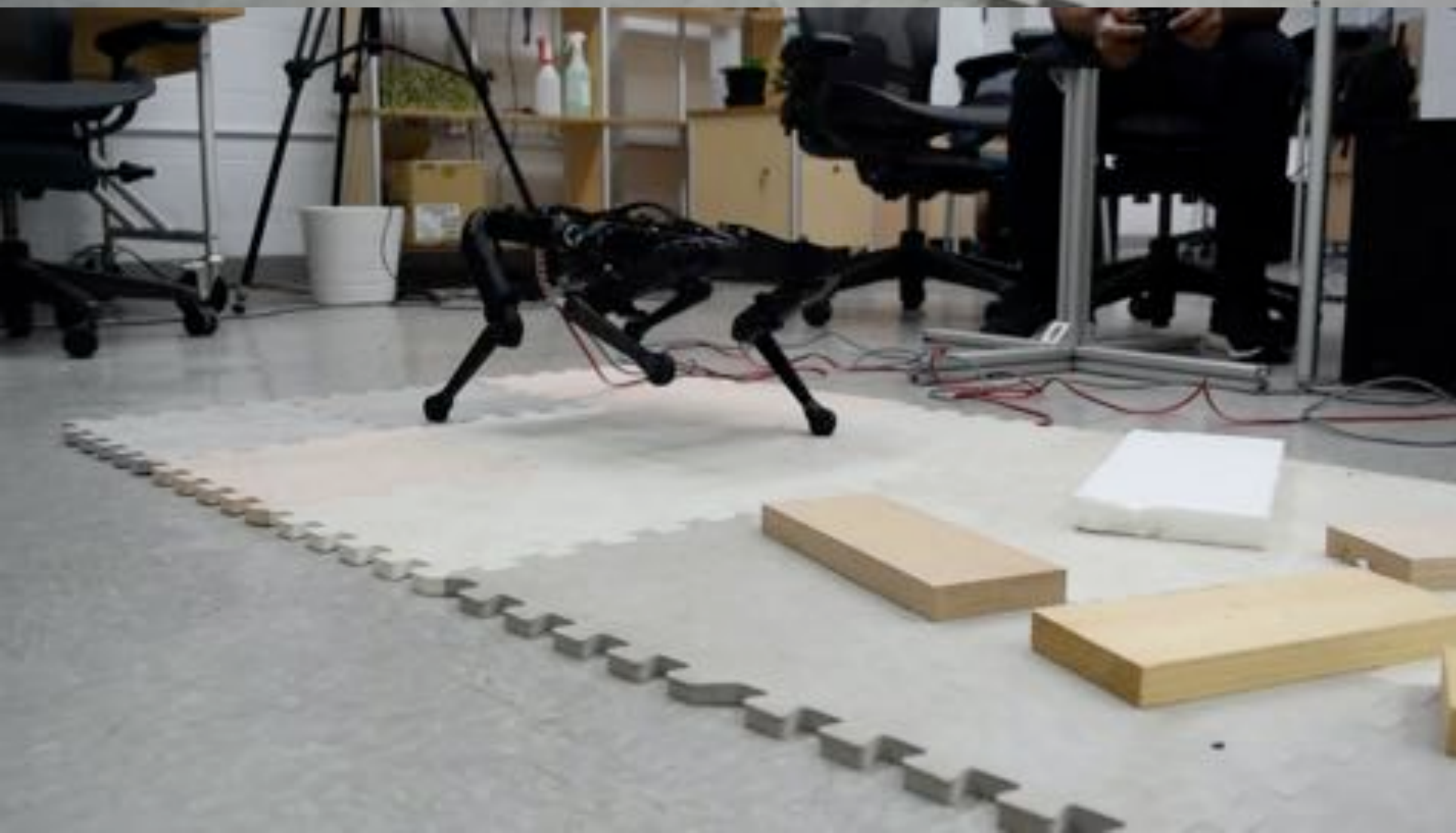


# Closed-loop nonlinear model predictive control



[Meduri et al., [arxiv.org/abs/2201.07601](https://arxiv.org/abs/2201.07601), T-RO in press]







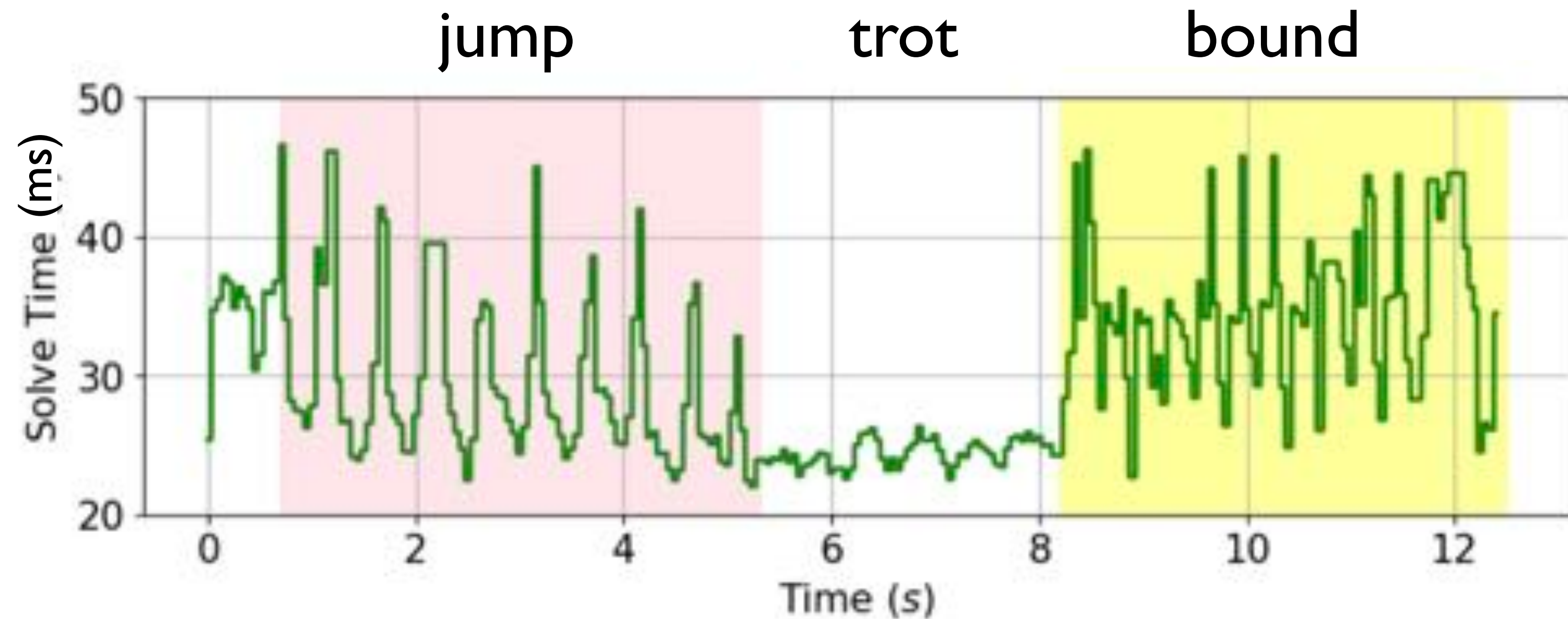




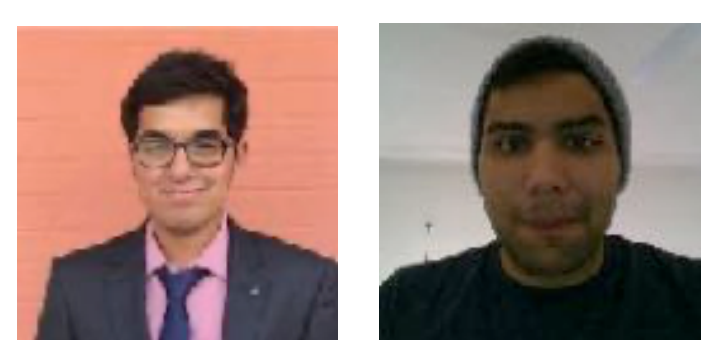




# Solve times (whole-body) during gait transitions

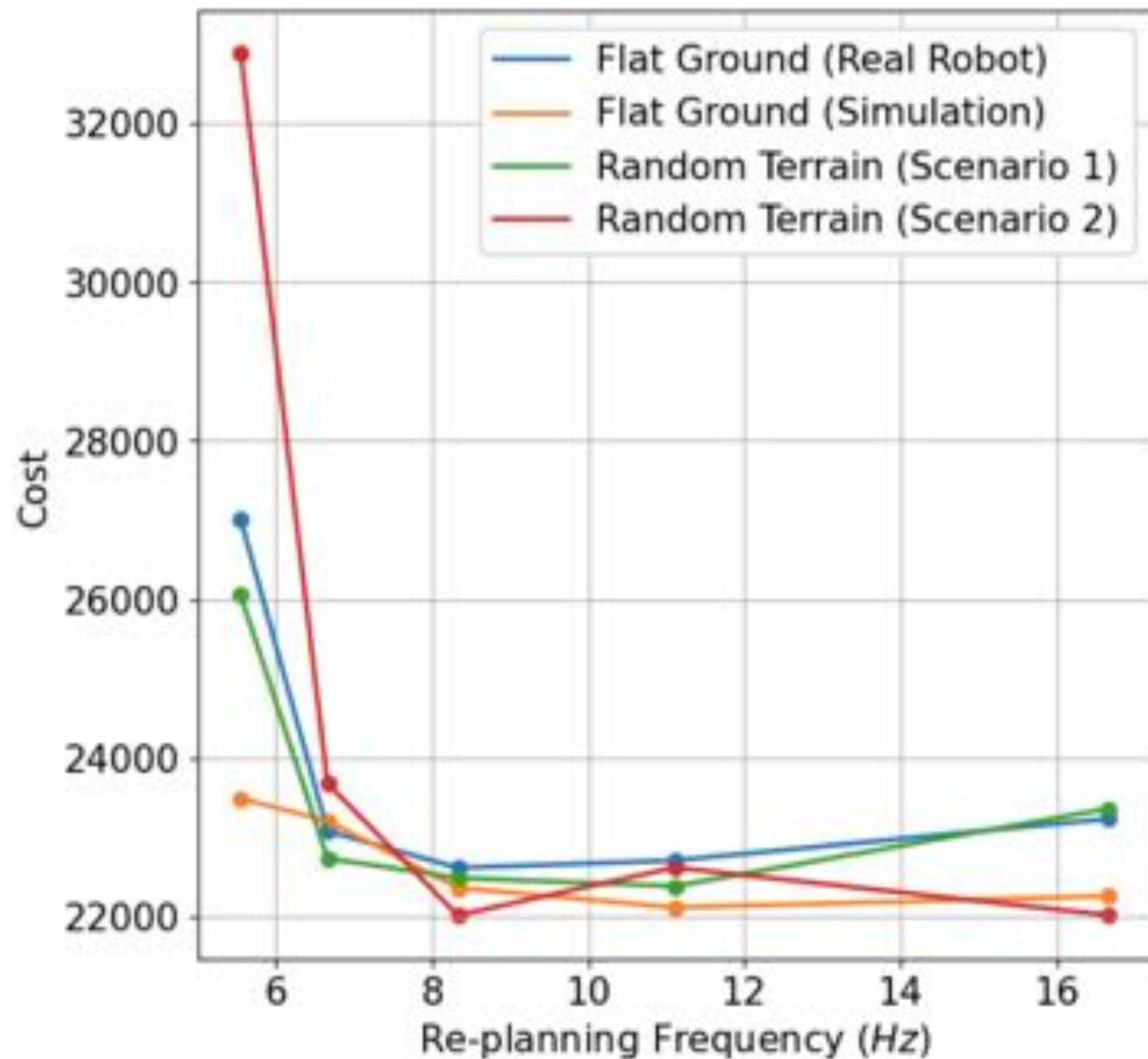


# of collocation points 10 (jump/trot) and 12 (bound)



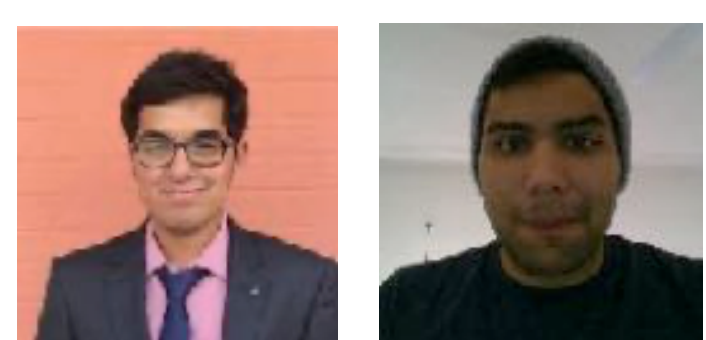
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# Do we need to close a faster MPC loop?



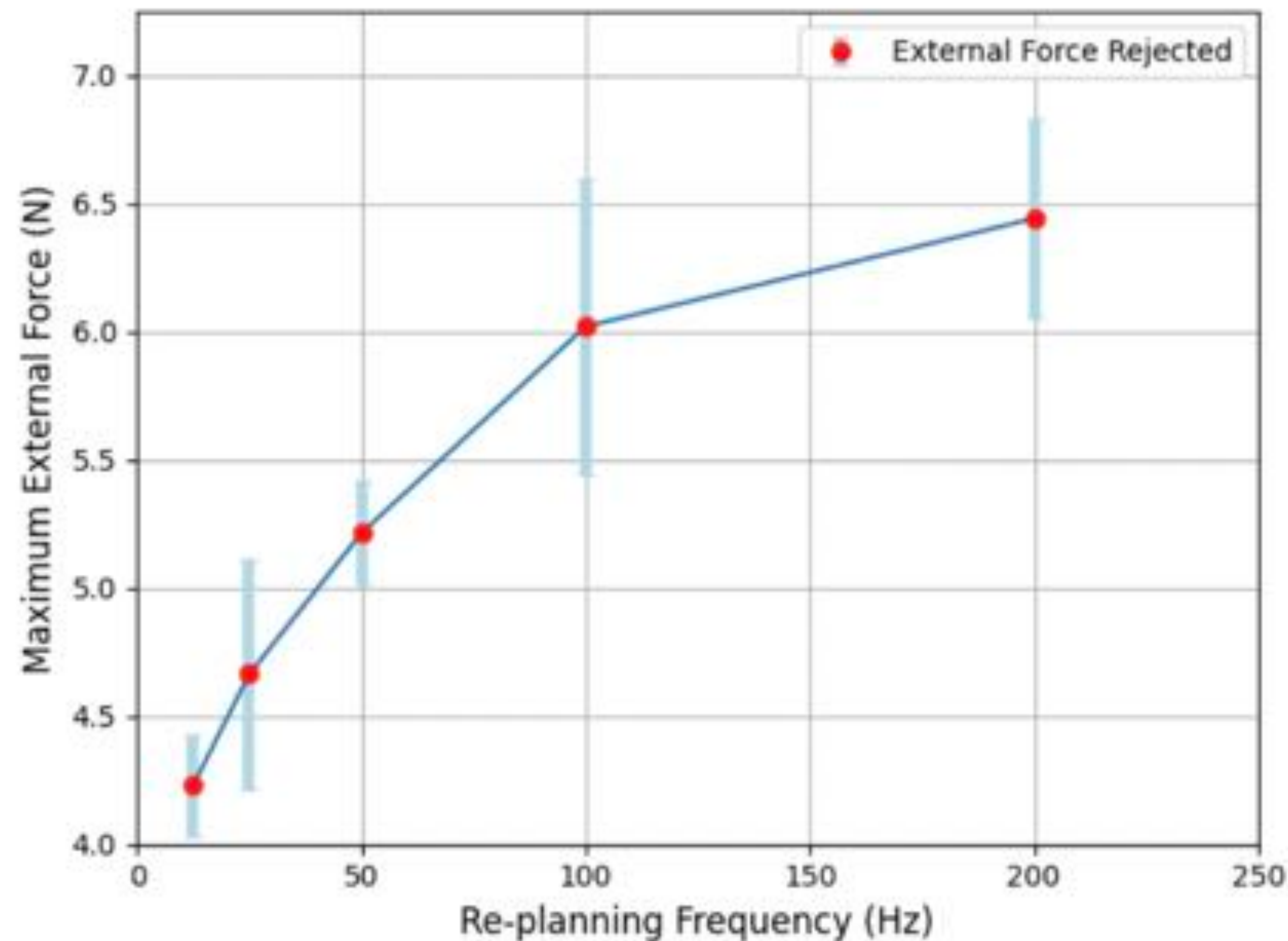
Measured cost as a function of re-planning frequency on 3 different terrains + real robot

Agreement real robot and simulation  
> 10Hz not much is gained



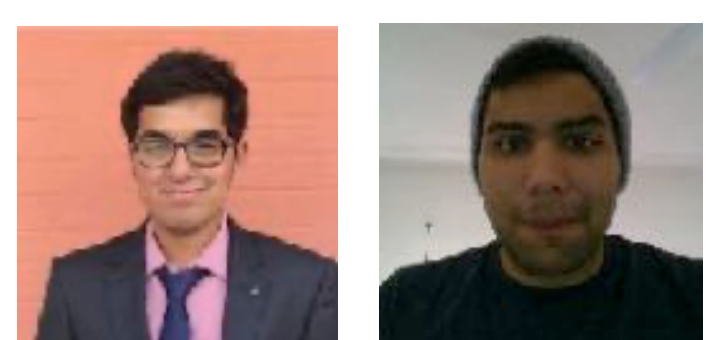
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# Do we need to close a faster MPC loop?



Maximum external force rejected as a function of re-planning frequency (simulation only)

Maximum disturbance rejection increases with MPC frequency



[Meduri et al., [arxiv.org/abs/2201.07601](https://arxiv.org/abs/2201.07601), T-RO in press]

# Closed-loop nonlinear model predictive control

- Efficient solver for whole-body MPC
- Closed-loop MPC leads to very robust behaviors (easy to transfer to the robot)
- First-order methods are very exciting

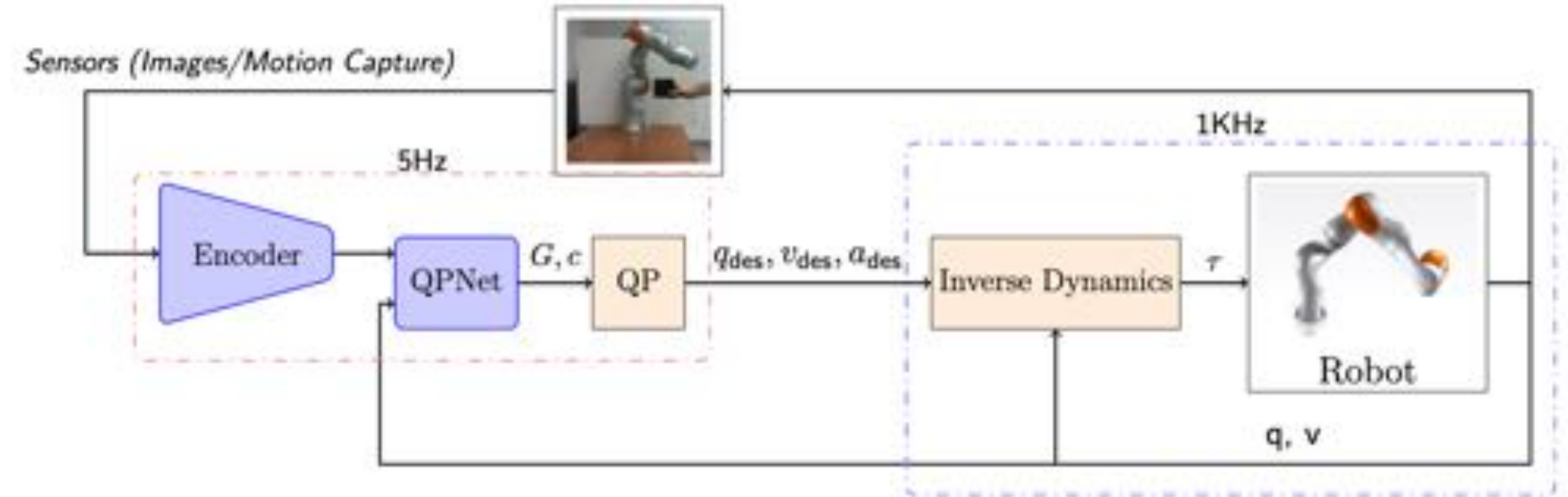
## Problems

- solve times are still high / requires a lot of online compute
- no online contact / gait adaptation
- high barrier to entry => need to write your own solver



Learning to reduce optimizer complexity

# Learning cost functions mapping sensors to OC problems



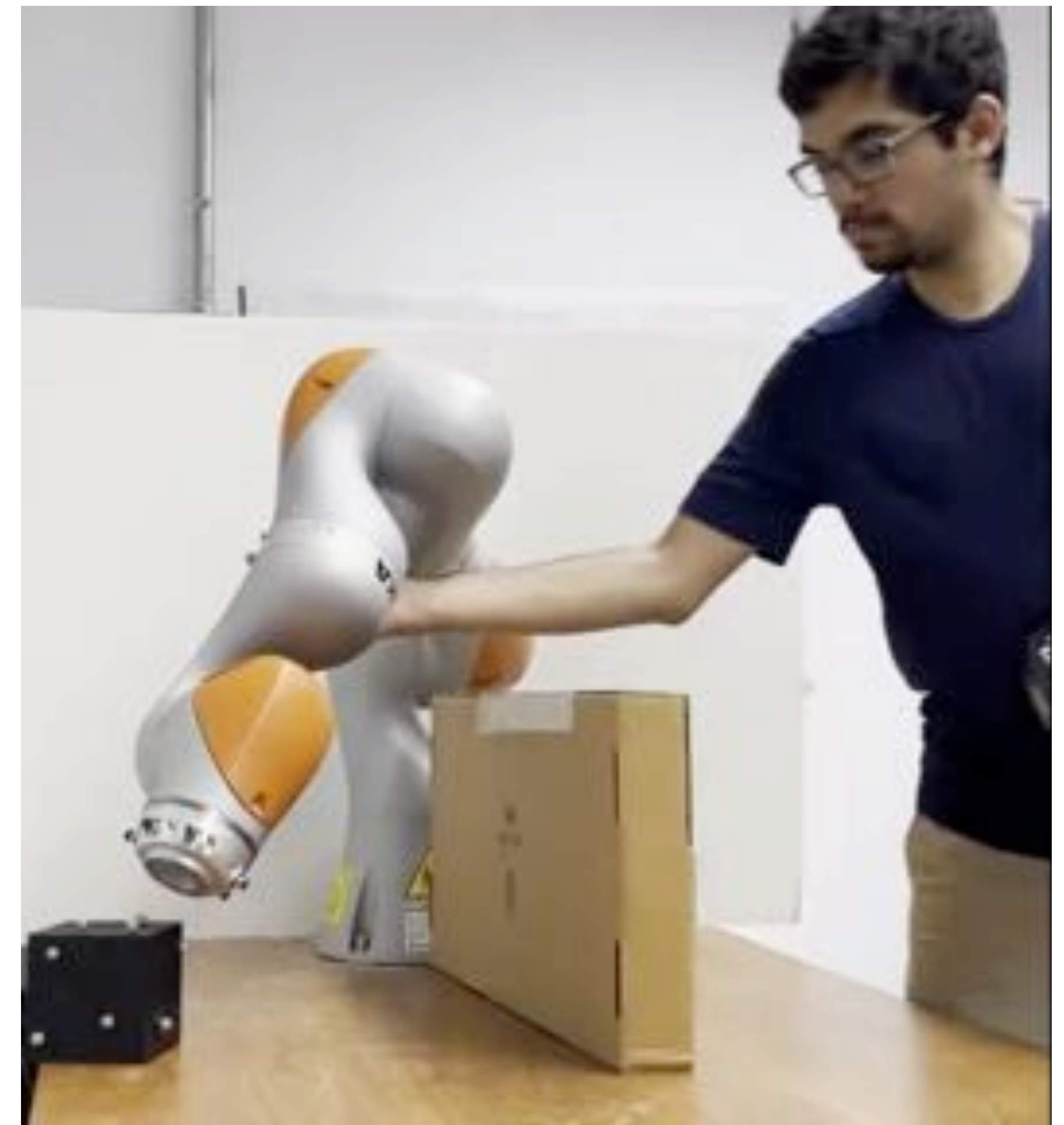
Move all the complexity in the cost function

Cost function NN  $\Rightarrow$  adapted sensor-driven quadratic cost at each control cycle

At each MPC cycle  $\Rightarrow$  simple QP with constraints



[Meduri et al., <https://arxiv.org/abs/2209.09451>]



Only a QP solved at each control cycle yet can avoid obstacles  
Cost includes vision + position/velocity sensing

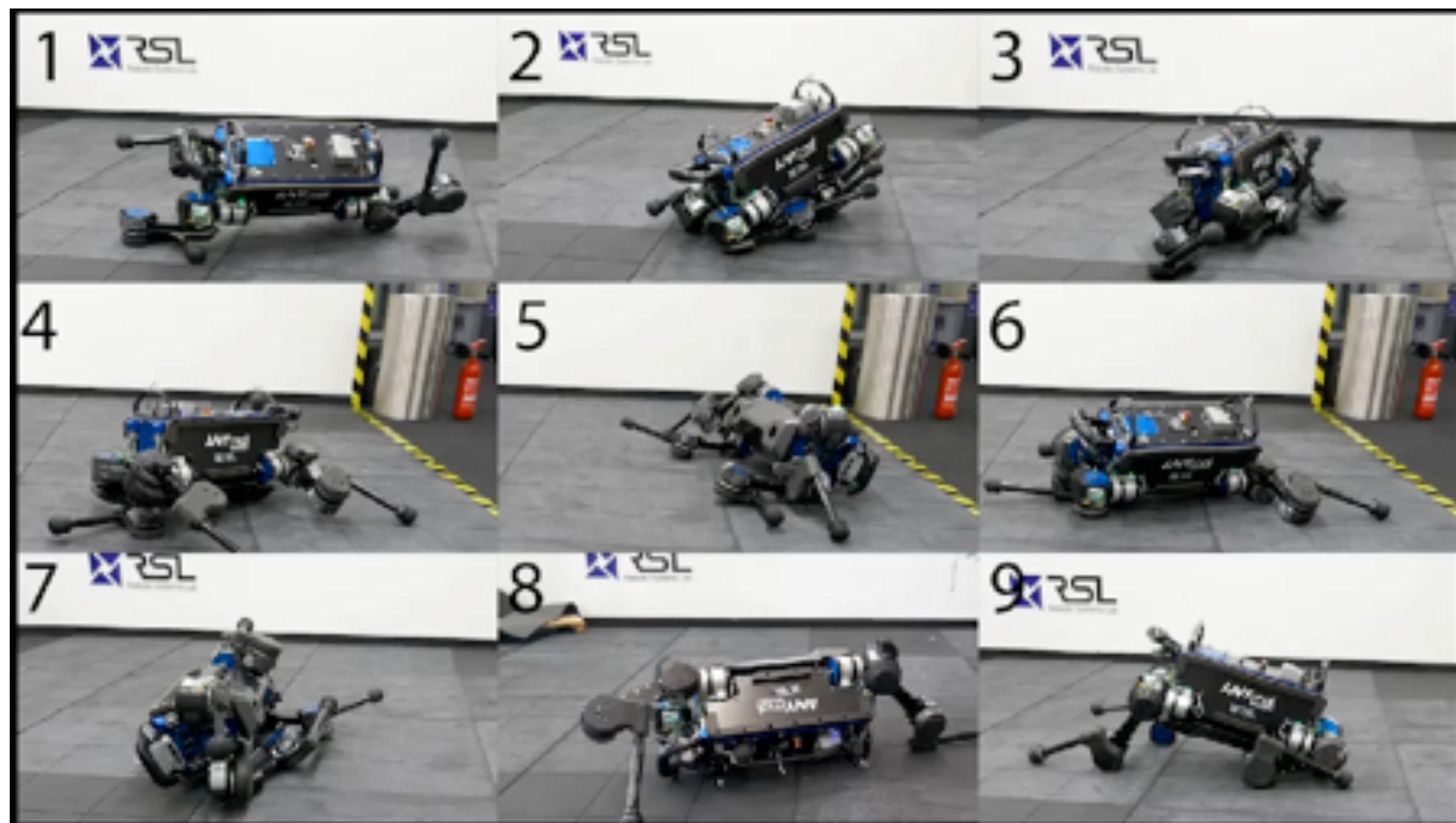


[Meduri et al., <https://arxiv.org/abs/2209.09451>]



But is this the way to go...?





[Hwangbo et al. 2019]



[Huang et al. 2022]

Direct policy learning with RL has had impressive successes

Perception is naturally included  
low barrier to entry (simple algorithms)

# Closed-loop nonlinear model predictive control

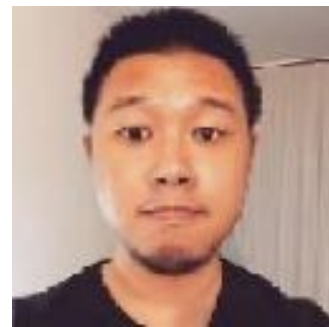
- Efficient solver for whole-body MPC
- Closed-loop MPC leads to very robust behaviors (easy to transfer to the robot)
- First-order methods are very exciting
- Learning costs / value function help lower complexity / include perception

Still many remaining issues:

- high barrier to entry for traj. opt. / MPC
- perception is often ignored
- where do we go from here? how do we relate to RL recent successes?

<https://github.com/machines-in-motion>





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